Final Exam - Solutions

Statistics - NYU, Summer 2016
Ercan Karadas

[1] a) (5 pt) 95% C.I. for the population mean μ :

$$\overline{X} \mp z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$2.9 \mp 1.96\frac{0.45}{\sqrt{25}}$$

$$\Longrightarrow [2.723,\ 3.076]$$

b) (7 pt)

$$2.99 - 2.90 = ME = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \Longrightarrow 0.09 = z_{\alpha/2}\frac{0.45}{\sqrt{25}}$$

Solving this gives $z_{\alpha/2} = 1$. Then $\alpha = 2[1 - F(1)] = 0.3174$, and therefore the confidence level is $100(1 - .3174)\% = 68.26\%$.

c) (7 pt) In the corrected data set the sample mean does not change but the number of observations reduces to 13. Therefore, a 95% C.I. for the population mean μ can be obtained just as in part (a) but with $n = 13$:

$$2.9 \mp 1.96\frac{0.45}{\sqrt{13}}$$

$$\Longrightarrow [2.655,\ 3.144]$$

d) (7 pt) A 95% C.I. for the population mean μ :

$$\overline{X} \mp t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}$$

$$2.9 \mp 2.064\frac{0.4}{\sqrt{25}}$$

$$\Longrightarrow [2.735,\ 3.065]$$

[2] a) (5 pt) $H_0 : \mu \geq 21.1$
$H_1 : \mu < 21.1$

b) (5 pt) Since $t_{24} = \frac{13.2-21.1}{3.7/5} = -10.68 < -t_{24,0.05} = -1.711$ reject H_0. There is sufficient evidence that filtered cigarettes have a mean tar amount less than 21.1 mg. The results suggest that the filters are effective in reducing the amount of tar.

c) (5 pt) We can not find an exact p-value from the t table, but from $t_{24} = -10.68$ we can say that $p - value < 0.001$.

d) (10 pt) $t_{24} = \frac{\overline{X}_c - 21.1}{3.7/\sqrt{25}} = -1.711 \Longrightarrow \overline{X}_c = 19.83$ and then
$\beta = P(t_{24} > \frac{19.83-17.98}{3.7/\sqrt{25}}) = P(t_{24} > 2.5) \approx 0.01$.
Therefore the power of the test is $1 - \beta \approx 99\%$

e) (7 pt) 90% C.I. for σ^2:

$$\frac{(24)(3.7)^2}{\chi^2_{24,0.05}} < \sigma^2 < \frac{(24)(3.7)^2}{\chi^2_{24,0.95}}$$
$$\implies 9 < \sigma^2 < 23.7$$

Since the C.I. interval doesn't contain 8, it is unlikely that the population variance σ^2 is 8.

[3] a) (7 pt) Let us define a random variable X_A to denote the length of time to complete an evaluation for a company's earnings forecast for analyst A. Then, we are given that $X_A \sim N(\mu_A, 1.7^2)$ and $\overline{X}_A = 6.4$ from a sample of size $n = 15$.
Therefore a 95% C.I. for the population mean μ :

$$\overline{X} \mp Z_{0.025}\frac{\sigma}{\sqrt{n}}$$
$$6.4 \mp 1.96\frac{1.7}{\sqrt{15}}$$
$$\implies [5.54,\ 7.26]$$

b) (7 pt) Since the probability of getting a sample mean below 6.4 is 0.67, we can write

$$P(\overline{X}_A < 6.4) = 0.67$$

Then transforming this into Z we obtain

$$\begin{aligned} 0.67 = P(\overline{X}_A < 6.4) &= P(\frac{\overline{X}_A - \mu_A}{\sigma/\sqrt{15}} < \frac{6.4 - \mu_A}{\sigma/\sqrt{15}}) && (1)\\ &= P\left(Z < \frac{6.4 - \mu_A}{1.7}\right) && (2)\\ &\implies \frac{6.4 - \mu_A}{1.7/\sqrt{15}} = 0.44 && (3)\\ &\implies \mu_A = 6.2 && (4) \end{aligned}$$

c) (7 pt) As in part (a) let us X_B to denote the length of time to complete an evaluation for a company's earnings forecast for analyst B. Then, we are given that $X_B \sim N(6.5, 2.3^2)$ and using the result in part (b) we also have $X_A \sim N(6.2, 1.7^2)$.
We can consider the next ten cases that they are going to work on as samples of size $n = 10$, and therefore the question is about sample averages $\overline{X}_A$ and $\overline{X}_B$, and in particular we would like to find the following probability:

$$P(\overline{X}_B - \overline{X}_A > 1) =?$$

But since $\overline{X}_A \sim N(6.2, \frac{1.7^2}{10})$ and $\overline{X}_B \sim N(6.5, \frac{2.3^2}{10})$, the distribution of their difference, $\overline{X}_B - \overline{X}_A$, will be

$$\overline{X}_B - \overline{X}_A \sim N(0.3, \frac{2.3^2 + 1.7^2}{10}) \implies \overline{X}_B - \overline{X}_A \sim N(0.3, 0.818)$$

Note that we used the information that completion times are independent, i.e. the covariance between the two random variables is zero. Now,

$$P(\overline{X}_B - \overline{X}_A > 1) = P(Z > \frac{1 - 0.3}{\sqrt{0.818}}) = P(Z > 0.774) = 1 - 0.7794 = 0.226$$

[4] (7 pt)

a) A 90% C.I. for the population variance:

$$\left[\frac{(n-1)s^2}{\chi^2_{14,0.05}}, \quad \frac{(n-1)s^2}{\chi^2_{14,0.95}}\right] = \left[\frac{(15-1)(16.7)}{23.685}, \quad \frac{(15-1)(16.7)}{6.571}\right] = [9.87, 35.58]$$

b) (7 pt)

$$\begin{aligned}
P(\sigma^2 < 11) &= P\left(\frac{1}{\sigma^2} > \frac{1}{11}\right) \\
&= P\left(\frac{(n-1)s^2}{\sigma^2} > \frac{(n-1)s^2}{11}\right) \\
&= P\left(\chi^2_{14} > \frac{(15-1)(16.7)}{11}\right) \\
&= P\left(\chi^2_{14} > 21.25\right) \\
&\approx 0.1
\end{aligned}$$

c) (7 pt)

$$\begin{aligned}
\text{Expected Profit} &= 520 \times P(\sigma^2 < 11) + 330 \times P(\sigma^2 > 11) \\
&\approx 520 \times (0.1) + 330 \times (0.9) \\
&= 349.
\end{aligned}$$