Problem Set 10

Statistics - NYU, Summer 2016
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Section 1

[1] Suppose annual returns on two stocks, R_1 and R_2, are normally distributed with the following distributions:

$$R_1 \sim N(0.1, 0.2^2) \quad \text{and} \quad R_2 \sim N(0.2, 0.4^2)$$

The covariance between the two stock returns is $Cov(R_1, R_2) = 0.03$. Consider an equal weighted portfolio with return

$$R_p = 0.5R_1 + 0.5R_2$$

a) What are the mean and variance of R_p (i.e. $E(R_p)$ and $Var(R_p)$).

b) Find the distribution of R_p?

c) Find the probability that weighted portfolio return will be higher than 0.329?

d) Suppose that there is another stock, R_3, that has the following normally distributed annual return and covariance with the second stock:

$$R_3 \sim N(0.1, 0.2^2) \qquad Cov(R_2, R_3) = -0.01$$

Would you replace R_1 with R_3 in the portfolio? Why?

[2] Scores on SAT test are normally distributed with a mean of 1518 and a standard deviation of 325. Scores on ACT test are also normally distributed with a mean of 21.1 and a standard deviation of 4.8. Assume that the two tests use different scales to measure the same aptitude.

a) If someone gets a SAT score that is the 67th percentile, find the actual SAT score?

b) For the same person in the previous part find the equivalent ACT score?

c) If someone gets a SAT score of 1900, find the equivalent ACT score?

d) If 25 ACT scores are randomly selected, find the probability that they have a mean greater than 20?

[3] A pharmaceutical manufacturer is concerned that the impurity concentration in pills should not exceed 3%. It is known that from a particular production run impurity concentrations follow a normal distribution. A random sample of 64 pills from a production run was checked, and the sample mean purity concentration was found to be 3.07%, and standard deviation is found to be 0.45%.

a) In the context of this problem, discuss whether one-sided or two-sided alternative hypothesis is more appropriate.

b) Test at the 5% level the appropriate (non-degenerate) null and alternative hypothesis.

c) Find the p-value of this test.

d) Assume that true impurity concentrations follow a normal distribution with mean 3.2% and standard deviation 0.45. Find the power of the test in part (b).

e) Suppose that the alternative hypothesis had been two-sided, rather than one-sided, with the null hypothesis $H_0 : \mu = 3$. State, without doing the calculations, whether the p-value of the test would be higher than, lower than, or the same as that found in part (c). Sketch a graph to illustrate your reasoning.

[4] Assume that the weight of cereal in a "10-ounce box" is $N(\mu, \sigma^2)$. To test $H_0 : \mu = 10.1$ against $H_1 : \mu > 10.1$, we take a random sample of size $n = 16$ and observe that $\bar{x} = 10.4$ and $s = 0.4$.

a) Do we reject or fail to reject the null hypothesis at the 5% significance level?

b) What is the approximate $p - value$ of this test?

[5] A company produces electric devices operated by a thermostatic control. The standard deviation of the temperature at which these controls actually operate should not exceed 2.0 degrees Fahrenheit. For a random sample of 20 of these controls the sample standard deviation of operating temperatures was 2.36 Fahrenheit. Stating any assumptions you need to make, test at 5% the null hypothesis that the population standard deviation is 2.0 against the alternative that is bigger.

[6] The manufacturer of a new product claims that his product will increase output per machine by at least 29 units per hour. A line manager adopts the product on 9 of his machines, and finds that the average increase was only 26 with a standard deviation of 6.2.

a) Formulate the appropriate null and alternative hypotheses to test whether there is sufficient evidence to doubt the manufacturer's claim?

b) Calculate the appropriate test statistic, and then conclude your test at 95% confidence level.

c) Calculate the appropriate p value? Is the result in the previous part statistically significant at the usual levels; namely 0.01, 0.05, and 0.10?

d) Now suppose that instead of 26, the sample mean is 31, how would you test the manufacturer's claim now? Just state the the appropriate null and alternative hypotheses.

Section 2

[7] A sample of 24 bottles is taken from the production line at a local bottling plant. Assume that the fill amounts follow a normal distribution.

a) What is the probability that the sample standard deviation is more than 80% of the population standard deviation?

b) The probability is 90% that the sample variance is less than what percent of the population variance?

c) Suppose that from the sample of 24 bottles, sample standard deviation is found to be 20. Does that information provide enough evidence that population standard deviation is greater than 15? State the hypothesis explicitly and conclude the test at $\alpha = 0.05$.

[8] In a test of the Atkins weight loss program, 41 individuals participated in a randomized trial with overweight adults. After 12 months, the mean weight loss was found to be 2.1 lb, with a standard deviation of 4.8 lb.

a) What is the best point estimate for the mean weight loss of all overweight adults who follow the Atkins program?

b) Construct a 90% confidence interval for the mean weight loss for all such subjects?

c) Test the hypothesis that the mean weight loss is 3.2 lb.

d) In part (c), could we conclude the test simply by looking at the confidence interval constructed in part (b)? Explain. Also, what is the minimum mean weight loss that would be rejected by the sample data?

e) Suppose that a weight loss program is considered effective only if the weight loss is at least 3.2 lb after 12 months. Do you think, the Atkinson program seems to effective at 90% confidence level?

f) For part (e), could we use the confidence interval constructed in part (b)?

[9] A simple random sample of 25 filtered cigarettes is obtained, and the tar content of each cigarette is measured. The sample has a mean of 13.2 mg and a standard deviation of 3.7 mg. We are interested in testing the claim that the mean tar content of filtered cigarettes is less than 21.1 mg, which is the mean for unfiltered cigarettes.

a) Formulate the appropriate null and alternative hypotheses.

b) Calculate the appropriate test statistic and conclude the test at $\alpha = 0.05$?

c) Find the p-value?

d) Find the power of the test given that true population mean tar content is $\mu^* = 17.98$.

e) Test the hypothesis that population variance is equal to 8 by constructing a 90% confidence interval.

[10] The marketing department for a hand calculator claims the battery pack performs 20,000 calculations before needing recharging. The quality control manager for the manufacturer is charged with validating the claim that the battery pack works as long as the specifications state in order to secure a large order from a key customer. A test of 30 battery packs yields an average of 19,597 calculations and a standard deviation of 1,103.

a) Formulate the appropriate null and alternative hypotheses?

b) Calculate the appropriate test statistic and conclude the test at $\alpha = 0.05$?

c) Calculate the appropriate p-value? Is the result in the previous part statistically significant at the usual levels; namely for α =0.01, 0.05, and 0.10?

d) Find the power of the test in part (b) given that $\mu = 19,310$.

e) Construct a 95% confidence interval for the population mean, and use this confidence interval to conclude the test stated in part (a)?

f) Based on the confidence interval you obtained in the previous part, can you conclude the test stated in part (a)? Briefly explain.