Problem Set 3 - Solutions

Statistics - NYU, Spring 2016
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[1]

a) $P_4^6 = \frac{6!}{(6-4)!} = 360$

b) $C_8^{10} = \frac{10!}{(10-8)! \cdot 8!} = 45$

c) $P_3^{26} \cdot P_3^{10}$

d) C_6^{44}

[2]

a) 4 books on fairy tales can be arranged in 4! ways. Similarly, 5 novels and 3 plays can be arranged in 5! and 3! ways, respectively. Therefore by the counting principle, all of them together can be arranged in $4! \times 5! \times 3! = 17280$ ways.

b) First, we consider the books on fairy tales, novels and plays as single objects. These three objects can be arranged in $3! = 6$ ways.

Let us fix one of these 6 arrangements. This may give us a specific order, say, novels - fairy tales - plays.

Given this order, the books on the same subject can be arranged as follows.
The 4 books on fairy tales can be arranged among themselves in $4! = 24$ ways.
The 5 novels can be arranged in $5! = 120$ ways.
The 3 plays can be arranged in $3! = 6$ ways.
For a given order, books can be arranged in $4! \times 5! \times 3! = 24 \times 120 \times 6 = 17280$ different ways.

Therefore, for all the 6 possible orders the books can be arranged in
$6 \times (4! \times 5! \times 3!) = 6 \times 17280 = 103680$ ways.

[3]

a) $\binom{16}{12} = \frac{16!}{4!12!} = 1,820$

b) # of women in the jury should be at least 7 to have a majority, therefore

$$\binom{8}{7}\binom{8}{5} + \binom{8}{8}\binom{8}{4}$$

[4] a) Sum of three integers is even if and only if either all of them are even or one is even and the other two integers are odds.

In the first case, we can choose three even integers from ten even integers in $\binom{10}{3}$ different ways, and zero odd integer from ten odd integers in $\binom{10}{0}$ different ways (note that $\binom{10}{0} = 1$), and therefore we can choose three even integers and zero odd integer in $\binom{10}{3}\binom{10}{0}$ different ways.

In the second case, we can choose two odd integers from ten odd integers in $\binom{10}{1}$ different ways, and one even integer from ten even integers in $\binom{10}{1}$ different ways, and therefore we can choose three two odd integers and one even integer in $\binom{10}{2}\binom{10}{1}$ different ways.

Therefore, in

$$\binom{10}{3}\binom{10}{0} + \binom{10}{2}\binom{10}{1} = \ldots = 570$$

different cases their sum will be even.

b) Their product is even unless all three numbers are odd numbers, or in other words if there is at least one even number then the product will be even. Therefore, the answer is

$$\binom{20}{3} - \binom{10}{3}\binom{10}{0} = \ldots = 1020$$

In words, we can choose three different numbers in $\binom{20}{3}$ different ways, and only $\binom{10}{3}\binom{10}{0}$ cases their product is odd. Therefore, their difference gives us the number of cases that their product is even.

[5] a) $S = \{HH, THH, HTHTHH, HTTTHH, \ldots\}$. The sample space has infinitely many elements. I have just listed a few of them. Important point is that each element in the sample space should end with two Heads and before these last two Heads, there should be no two successive Heads in the previous rounds.

b) There are only two cases that it will be tosses four times: $TTHH$ and $HTHH$. Therefore, this event can written as $E_1 = \{TTHH, HTHH\}$

c) $E_2 = \{HH, THH, TTHH, HTHH\}$

d) $E_3 = \{THH, TTHH, HTHH, TTTHH, THTHH, HTTHH\}$

[6] a) $S = \{(1, 1; 1, 1), (1, 1; 1, 2), \ldots, (2, 1; 1, 1), (2, 1; 1, 2), \ldots, (3, 3; 3, 3)\}$

b) $E_1 = \{(1, 1; 1, 1), (1, 1; 1, 2)\}$ $\qquad E_2 = \{(2, 3; 1, 1), (2, 3; 3, 2)\}$

c) $S = \{(3,1),(3,2),(1,3),(2,3),(1,1;1,2),\ldots,(2,1;1,1),\ldots,(3,3;3,3)\}$
For example S can not include elements like (3,1;1,1) or (2,3;1,1), because under the given condition in both of these cases the game would not go the second round.

[7] a) C_4^6

b) As there are 10 non-red, we choose 4: C_4^{10}

c) There are three possibilities: (2R, 1W, 1B), (1R, 2W, 1B) or (1R, 1W, 2B):

$$C_2^6 C_1^7 C_1^3 + C_1^6 C_2^7 C_1^3 + C_1^6 C_1^7 C_2^3$$

[8] a) Similarly to question 5, the sample space is infinite. $S = \{H, TH, TTTH, TTTTTH, \ldots\}$. The sample space always end on heads.

b) This event can only happen in such a way only: $E = \{TTH\}$

[9] Let 4 girls be one unit and now there are 6 units in all. They can be arranged in 6! ways. In each of these arrangements 4 girls can be arranged in 4! ways. Therefore, total number of arrangements in which girls are always together is equal to $6! \times 4!$

[10]

Part A	Part B	# of Ways
2	4	$C_2^5 C_4^5$
3	3	$C_3^5 C_3^5$
4	2	$C_4^5 C_2^5$

Therefore, the candidate can select the questions in $C_2^5 \cdot C_4^5 + C_4^5 \cdot C_2^5 + C_3^5 \cdot C_3^5$ different ways.

[11] a) There are three cases: (2W, 3M),(3W,2M), (4W,1M), and therefore
$C_2^4 \cdot C_3^6 + C_3^4 \cdot C_2^6 + C_4^4 \cdot C_1^6$

b) $C_0^4 \cdot C_5^6 + C_1^4 \cdot C_4^6 + C_2^4 \cdot C_3^6$

[12] It is assumed that the consonants and vowels all have to be distinct. From 5 consonants, 3 consonants can be selected in C_3^5 ways. From 4 vowels, 2 vowels can be selected in C_2^4 ways. Now with every selection, number of ways of arranging 5 letters is 5!. Therefore, total number of words is $C_3^5 \cdot C_3^4 \cdot 5!$

[13] In an analogous way to the previous exercise the total number of words is $C_2^5 \cdot C_3^{21} \cdot 5!$

[14] a) $C_2^3 \cdot C_4^6 \cdot C_3^4 \cdot 9!$

b) $C_2^3 \cdot C_4^6 \cdot C_3^4 \cdot 2! \cdot 4! \cdot 3! \cdot 3!$

c) $C_2^3 \cdot C_4^6 \cdot C_3^4 \cdot 2! \cdot 4! \cdot 3!$