Problem Set 4 - Solutions

Statistics - NYU, Summer 2016
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[1] a) Since each outcome is equally likely this probability is simply equals to

$$\frac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in } S} = \frac{2}{10}$$

Alternatively, note that there are 2 mutual exclusive ways this can happen: two ones, two twos. The sum of the corresponding probabilities is

$$\frac{\binom{2}{2}\binom{3}{0} + \binom{2}{2}\binom{3}{0}}{\binom{5}{2}} = \frac{2}{10}$$

For example, the first term in the denominator says there are $\binom{2}{2}\binom{3}{0}$ different ways to choose two "ones".

b)

$$\frac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in } S} = \frac{4}{10}$$

Alternatively, note that there are 2 mutual exclusive ways this can happen: two reds, two blues. The sum of the corresponding probabilities is

$$\frac{\binom{3}{2}\binom{2}{0} + \binom{2}{2}\binom{3}{0}}{\binom{5}{2}} = \frac{4}{10}$$

c)

$$\frac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in } S} = \frac{6}{10}$$

Alternatively, note that there are 4 mutual exclusive ways this can happen: two reds, two blues, two ones, two twos. The sum of the corresponding probabilities is

$$\frac{\binom{3}{2}\binom{2}{0} + \binom{3}{0}\binom{2}{2} + \binom{2}{2}\binom{3}{0} + \binom{2}{2}\binom{3}{0}}{\binom{5}{2}} = \frac{6}{10}$$

[2] a) Four red chips can be chosen in $\binom{6}{4}$ different ways, and in total four chips can be chosen in $\binom{16}{4}$ ways, therefore probability of choosing four red chips is $\frac{\binom{6}{4}}{\binom{16}{4}}$.

b) Since there are ten non-red chips, we can choose four chips out of them in $\binom{10}{4}$ ways, and therefore probability that none of the four chips is red given by $\frac{\binom{10}{4}}{\binom{16}{4}}$.

c) There are three possibilities: (2R, 1W, 1B), (1R, 2W, 1B) or (1R, 1W, 2B). Since these events are mutually exclusive we need to sum up probabilities of each of these three events to find the answer:

$$\frac{\binom{6}{2}\cdot\binom{7}{1}\cdot\binom{3}{1}}{\binom{16}{4}} + \frac{\binom{6}{1}\cdot\binom{7}{2}\cdot\binom{3}{1}}{\binom{16}{4}} + \frac{\binom{6}{1}\cdot\binom{7}{1}\cdot\binom{3}{2}}{\binom{16}{4}}$$

[3] There are only two cases that it will be tossed exactly four times; (T, T, H, H) and (H, T, H, H). Therefore,

$$P(4 \text{ tosses}) = P\big((T, T, H, H) \text{ or } (H, T, H, H)\big) = \frac{1}{2^4} + \frac{1}{2^4} = \frac{1}{8}$$

[4] Using Bayes' rule:

$$P(A_1|B_1) = \frac{P(A_1)P(B_1|A_1)}{P(A_1)P(B_1|A_1) + P(A_2)P(B_1|A_2)} = \frac{0.4 \times 0.6}{0.4 \times 0.6 + 0.6 \times 0.7} = 0.36$$

[5] S=Capital punishment supporter
F=Female
P(S)=0.64
P(F)=0.48
$P(S|F)$=0.46

a) $P(S \cap F) = P(S|F)P(F) = (0.46)(0.48) = 0.2208$

b) M=Male Since $P(S) = P(S \cap F) + P(S \cap M)$, we have $0.64 = 0.2208 + P(S \cap M)$, then $P(S \cap M) = 0.4192$

c) Since $P(M) = P(M \cap S) + P(M \cap \bar{S})$, we have $0.52 = 0.4192 + P(M \cap \bar{S})$. Then, $P(M \cap \bar{S})$= 0.1008.

d) $P(M|\bar{S}) = P(M \cap \bar{S})/P(\bar{S}) = 0.1008/0.36 = 0.28$

[6] M: Men
F: Women
G: Graduate training
UG: Undergraduate training

HS: High school training

Then let us write what we are given in the problem: $P(M) = .8$, $P(F) = .2$, $P(G|M) = .1$, $P(HS|M) = .6$, $P(UG|M) = .3$, $P(G|F) = .15$, $P(UG|F) = .4$, $P(HS|F) = .45$

a) $P(M \cap HS) = P(HS|M)P(M) = (.6)(.8) = .48$

b) $P(G) = P(M \cap G) + P(F \cap G) = P(G|M)P(M) + P(G|F)P(F) = (.10)(.8) + (.15)(.2) = .11$

c) $P(M|G) = P(M \cap G)/P(G) = .08/.11 = .7273$

d) No, since $P(M \cap G) = .8$ is not equal to $P(M)P(G) = .88$.

e) $P(F|\bar{G}) = P(F \cap \bar{G})/P(\bar{G}) = \frac{[P(F)-P(F \cap G)]}{1-P(G)} = \frac{(.2-(.15)(.2))}{.89} = .191$

[7] **Solution 1.**

DD: Disk drive errors

CM: Computer memory errors

OS: Operating system errors

F: Failure.

$P(DD) = 0.50$, $P(CM) = 0.30$, $P(OS) = 0.20$; $P(F|DD) = 0.60$, $P(F|CM) = 0.70$, $P(F|OS) = 0.30$.

We want to find $P(DD|F)$=?

From Bayes's rule:

$$\begin{aligned} P(DD|F) &= \frac{P(F|DD)P(DD)}{P(F|DD)P(DD) + P(F|CM)P(CM) + P(F|OS)P(OS)} \\ &= \frac{(0.60)(0.5)}{(0.60)(0.5) + (0.70)(0.3) + (0.3)(0.2)} \\ &= 0.53 \end{aligned}$$

Solution 2. Geometric Approach as solved in the recitation.

[8] a) A: Alice hits, B: Beatriz hits, H: target is hit (i.e., either by A or by B, or both)

$$\begin{aligned} P(B|H) &= \frac{P(B,H)}{P(H)} \\ &= \frac{P(B)}{P(H)} \\ &= \frac{P(B)}{1-P(\bar{H})} \\ &= \frac{0.4}{1-(0.3)(0.6)} \\ &= \frac{20}{41} \end{aligned}$$

Note that $P(B, H) = P(B \text{ hits and the target is hit}) = P(B \text{ hits})$.

b) Let us first find the probability that exactly one of them will hit. There are two cases: either (A and not B) or (B and not A). Therefore, probability of exactly one hit is $P(1hit) = P(A, \bar{B}) + P(B, \bar{A}) = (0.7)(0.6) + (0.4)(0.3) = 0.54$. Now we want to find $P(B|1hit)$:

$$\begin{aligned} P(B|1hit) &= \frac{P(B, 1hit)}{P(1hit)} \\ &= \frac{P(B, \bar{A})}{P(1hit)} \\ &= \frac{(0.4)(0.3)}{1 - (0.3)(0.6)} \\ &= \frac{2}{9} \end{aligned}$$

Note that $P(B, 1hit)$ means B hits and there will be exactly one hit, which is equivalent to say B hits and A does not, which in turn is equal to $P(B, \bar{A})$. We used this observation in the calculations above.

[9] G: Eventually graduate
F: Freshmen
JC: Enters as community college transfer.

a) $P(G \cap F) = P(G|F)P(F) = (.62)(.73) = .4526$.
b) $P(G) = P(G|F)P(F) + P(G|JC)P(JC) = .4526 + (.78)(.27) = .663$.
c) $P(G \cup F) = P(G) + P(F) - P(G \cap F) = .6632 + .73 - .4526 = .9406$
d) No, since $P(G \cap JC)$ which is $(.78)(.27) = .2106 \neq .1791$ which is P(G)P(JC) $= (.6632)(.27)$

[10] W: customer orders Wine
R: customer is regular
O: customer is occasional
N: customer is new

a) $P(W) = P(W|R)P(R) + P(W|O)P(O) + P(W|N)P(N) = (.7)(.5) + (.5)(.4) + (.3)(.1) = .58$
b) $P(R|W) = P(R \cap W)/P(W) = .35/.58 = .6034$
c) $P(O|W) = P(O \cap W)/P(W) = (.5)(.4)/0.58 = 0.3448$

[11] C: The patient has been cured
D: The patient has been given the drug.
Given: $P(C|\bar{D}) = 0.5$ and $P(C|D) = 0.75$

a) As we expect 7.5 patients out of the 10 patients to be cured, out of the total pool of people, we will have 7.5% of such patients.

b) In the remaining pool of 90 people, 45 will be cured. Adding the 7.5 there is a total pool of cured people of 52.5. Then the probability of being cured and given the drug is 7.5/52.5 = 0.142 = 14.2%. Using Bayes' Rule:

$$P(C|D) = \frac{P(C \cap D)}{P(C)} = \frac{0.142}{0.525} = 0.270 = 27\%$$

c) As we have a specific group of 10, we can compute the probability of picking them out. In the first pick, we have 10/100 probability to get the correct person. Then 9/99 and so on. We can write this as:

$$\frac{10}{100} \times \frac{9}{99} \times .... \times \frac{1}{91} = \frac{10!}{\frac{100!}{90!}} = \frac{10!90!}{100!}$$

[12] G: The store has been rated good.
S: The store has been successful.
Given: $P(S) = 0.6$,$P(G|S) = 0.7$ and $P(G|\bar{S}) = 0.2$

a) $P(G) = P(G \cap S) + P(G \cap \bar{S}) = 0.6 \times 0.7 + 0.4 \times 0.2 = 0.5 = 50\%$

b) $\frac{P(G \cap S)}{P(G \cap S) + P(G \cap \bar{S})} \frac{0.7 \times 0.6}{0.5} = 0.84 = 84\%$

c) No. As $P(G|S) \neq P(G)$ or

d) Take the complement that no stores chosen are successful. So $1 - P(\bar{S})^5 = 1 - (0.4)^5 = 0.9876 = 98.8\%$

[13] R: field using regular plowing
H: high yield
Given that: $P(H|\bar{R}) = 0.5$ and $P(\bar{H}|R) = 0.4$ and P(R)=0.4
So:
$P(H|R) = 1 - P(\bar{H}|R) = 0.6$ and using Bayes' Rule:

$$P(R|H) = \frac{0.6 \times 0.4}{0.6 \times 0.4 + 0.5 \times 0.6} = 0.444$$

[14] M: Has MBA
O: over 35 Given: $P(M) = 0.35$, $P(O) = 0.4$ and $P(O|M) = 0.35$.

a) $P(M)P(O|M) = 0.3 \times 0.35 = 0.105$

b) $P(M|O) = \frac{P(O \cap M)}{P(O)} = \frac{0.105}{0.4} = 0.26$

c) $P(M) + P(O) - P(M \cap O) = 0.35 + 0.4 - 0.105 = 0.645$

d)

$$P(\bar{M}|O) = \frac{P(\bar{M})P(O|\bar{M})}{P(\bar{M})P(O|\bar{M}) + P(M)P(O|M)} = \frac{0.75 \times 0.7}{0.75 \times 0.7 + 0.35 \times 0.3} = 0.833$$

e) $P(O \cap M) = 0.105$, while $P(M)P(O) = 0.35 \times 0.4 = 0.1399$ so not independent.

f) No, as both event do happen.

g) No, by part c. There can be people under 35 without MBA's.