Problem Set 6

Statistics - NYU, Summer 2016
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Section 1

[1] The number of people arriving at a bicycle repair shop follows a Poisson distribution with an average of 5 arrivals per hour. Let X be the number of people arriving per hour.

a) What is the probability that seven people arrive at the bike repair shop in a one hour period of time?

b) What is the probability that at most seven people arrive at the bike repair shop in a one hour period of time?

c) What is the probability that more than seven people arrive at the bike repair shop in a one hour period of time?

d) What is the probability that between 4 and 9 people, inclusively, arrive at the bike repair shop in a one hour period of time?

[2] Delta International delivers approximately one million packages a day between East Asia and the United States. A random sample of the daily number of package delivery failures over the past six months provided the following results: 15, 10, 8, 16, 12, 11, 9, 8, 12, 9, 10, 8, 7, 16, 14, 12, 10, 9, 8, 11. There was nothing unusual about the operations during these days and, thus, the results can be considered typical. Using these data and your understanding of the delivery process answer the following:

a) What probability model should be used and why?

b) What is the probability of 10 or more failed deliveries on a typical future day?

c) What is the probability of less than 6 failed deliveries?

[3] Suppose that the random variable X has the following probability distribution function

X	-2	1	2
P(X)	.5	.3	.2

Construct the probability distribution tables for the following random variables

a) $Y_1 = 3 - 7 \cdot X$

b) $Y_2 = 0.5 \cdot X$

c) $Y_3 = X^2 + X - 2$

d) $Y_4 = X^2$

[4] Compute the expected value and variance for the random variables defined in the previous problem.

[5] Consider a discrete random variable X. In contrast to the previous problem, suppose that the probability distribution function of X is not explicitly given. All you know is that $\mu_x = 3$ and $\sigma_x^2 = 1$.

Discuss whether the expected value and the variance can be calculated for each of the random variables defined below. If yes, provide the answer.

a) $Y_1 = 3 - 7 \cdot X$

b) $Y_2 = 0.5 \cdot X$

c) $Y_3 = X^2 + X - 2$

d) $Y_4 = X^2$

[6] Consider the joint probability distribution:

$X(row)/Y$	0	2
1	.30	.20
3	.25	.25

a) Compute the marginal probability distributions for X and Y.

b) Compute the covariance and correlation for X and Y.

c) Are they independent?

d) Compute the mean and variance for the linear function $W = 2X + Y$.

e) Compute the mean and variance for the random variable $W_3 = X^2 + Y$.

[7] Suppose you were asked to analyze a stock portfolio that contains 5 shares of stock A and 10 shares of stock B. The joint probability distribution of the stock prices is presented in the table below. For example, $P(A = \$60, B = \$55) = 0.20$, and so on. Also, numbers at the end of each row and column denote the marginal probabilities, i.e., $P(B = \$55) = 0.50$ etc.

$A(row)/B$	\$35	\$55	\$80	Marg. of A
\$50	.10	?	?	.30
\$60	.10	.20	?	.40
\$70	?	.15	.10	?
Marg. of B	?	.50	.25	1.00

a) Complete the missing joint and marginal probabilities in the table.

b) Compute the expected value of the portfolio.

c) Compute the standard deviation of the portfolio.

d) What is the expected price of Stock A given that price of Stock B is 55?

Section 2

[8] More than 50 million guests stay at bed and breakfast (B&Bs) each year. The website for the Bed and Breakfast Inns of North America, which averages seven visitors per minute, enables many B&Bs to attract guests.

a) What is the probability that exactly three will visit the website in a one-minute period?

b) What is the probability that at most two will visit the website in a one-minute period?

c) What is the probability that exactly ten will visit the website in a five-minute period?

d) Suppose that 10% of the web visitors calls Inns of North America in the next ten minutes for reservation. Assume that if visitors don't call in ten minutes after they check the website, they will never call. What is the probability that at any given moment, the company will receive 3 calls in the next ten minutes from the web visitors?

[9] In the lecture, we said 'if X is a random variable any linear function of X is also a random variable'. That statement was not futile. Can you think of an example where X is a discrete random variable, but some function of it is not random anymore?

[10] Consider jointly distributed two random variables X and Y, with $\mu_x = 0.7$, $\sigma_x^2 = 0.04$ and $\mu_y = 1.1$, $\sigma_y^2 = 0.09$. Also their covariance is given as -0.03.

Discuss whether the expected value and the variance for the random variables defined below can be computed. If yes, provide the answer.

a) $W_1 = 3X + Y$

b) $W_2 = Y - 0.5X$

c) $W_3 = X^2 + Y$

d) Discuss why you were able to compute μ_{w3} and σ^2_{w3} in 6.e, but not in part c here.

[11] The following table displays the joint probability distribution of two discrete random variables X and Y.

$Y(row)/X$	1	2	3
0	.10	.12	.06
1	.05	.10	.11
2	.02	.16	.28

a) Determine the marginal probability distribution for X.

b) Compute the expected value for X, $\mu_X =?$

c) Compute the standard deviation for X, $\sigma_X =?$

d) Compute the covariance between X and Y, $Cov(X, Y) =?$

e) Compute the expected value for X given that $Y = 2$, $E(X|Y = 2) =?$

f) For $W = 2X + Y$ compute the expected value, $E(W) =?$

g) Compute $E(W|X + Y = 3)$: the expected value of $W = 2X + Y$ given that $X + Y = 3$.

[12] The joint probability distribution of two random variables C and R is given in the table below, where C is the daily total cost and R is the daily total revenue for a company operating in manufacturing sector. For example in the table, with 0.20 probability cost is \$14 and revenue is \$10, so the company makes \$4 loss.

$C(row)/R$	\$10	\$15	\$20
\$12	.15	.10	.05
\$14	.20	.15	.10
\$16	.05	.05	.15

a) Determine the marginal probability distributions of C and R.

b) What is the expected daily cost of the company, E(C)=?

c) What is the expected daily profit of this company? (Profit (π) is just the difference between R and C, i.e $\pi = R - C$.)

d) For randomly chosen day, given that the firm makes a loss, what is the probability that the loss is \$6?

e) For randomly chosen three days, what is the probability that the firm makes loss in two of them.

f) For a randomly chosen day, find the expected profit of the company given that its cost is \$14 in that day, $E(\pi|C = 14) =?$