Review of Probability (SW Ch. 2)

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Outline

The Fundamental Concepts of Probability

We will review the fundamental concepts of probability through examples

- **Experiment:** Tossing an *unbiased* coin 3 times.
- The **sample space**:

$$S = \{HHH, TTT, HHT, THH, HTH, HTT, THT, TTH\}$$

- A **Random Variable**:

$$X = \# \text{ of Heads in 3 coin tossings}$$

$$\implies X \in \{0, 1, 2, 3\}$$

- The **Probability Distribution** of X:

X	$P(X = x)$
0	1/8
1	3/8
2	3/8
3	1/8

- Notation: Capital letters are used to denote r.v.s and small letters to denote particular values that a random variable can assume.

- $P(X)$ is called the **Probability Distribution Function (pdf)** of X
- Variables like X can take on different values on each trial of the experiment. Since we cannot tell which value will be assumed in a particular trial, we call them **random variables**.
- In general, a random variable assigns a numerical value to each possible realization of an experiment.
- Whenever we have a r.v. X, we should think about two objects:
  - The possible values of X,
  - The probability distribution function of X
- If a random variable can take on no more than a countable number of values, we call it a **Discrete Random Variable**, whereas a **Continuous Random Variable** can take on a continuum of possible values.
- The **Cumulative Probability Distribution** of X

X	$P(X = x)$	$\mathbf{P(X \leq x)}$
0	1/8	**1/8**
1	3/8	**4/8**
2	3/8	**7/8**
3	1/8	**1**

- Let us consider the same experiment with a *biased* coin:

$$P(H) = 0.4 \text{ and } P(T) = 0.6$$

- Let us construct the probability distribution table again

X	$P(X = x)$
0	$(0.6)^3$
1	$3(0.4)(0.6)^2$
2	$3(0.4)^2(0.6)$
3	$(0.4)^3$

- The probability distribution of the same r.v. X depends on the **Population Distribution** of Heads (and Tails): the fraction of coins with Heads up in urn containing large number of (infinitely many) coins.

Example

- Let M be the number of times your computer crashes while you are writing a term paper with the following probability distribution:

	Outcome (number of crashes)				
	0	1	2	3	4
$P(M = m)$	0.80	0.10	0.06	0.03	0.01
$P(M \leq m)$	0.80	0.90	0.96	0.99	1.00

Table: Population Distribution of Crashing Times

- Some computations:
  - $P(M = 0) = 0.80$: probability of no computer crashes
  - $P(M = 1 \text{ or } M = 2) = P(M = 1) + P(M = 2) = 0.16$: probability that of one or two computer crashes will take place
  - $P(M \leq 2) = P(M = 0) + P(M = 1) + P(M = 2) = 0.96$: probability of at most two crashes

Example

- In a classroom with 35 females and 65 males, what is the probability of having 3 females in a randomly selected group of 5 (with replacement)?
- Let F : # of females selected in the experiment
- A succinct way of expression the probability distribution of F

$$P(F = f) = \binom{5}{f}(0.35)^f(0.65)^{5-f}$$

- Therefore, we compute the desired probability as

$$P(F = 3) = \binom{5}{3}(0.35)^3(0.65)^2$$

Expected Value and Variance of a Discrete R.V.

Consider a discrete r.v. X with p.d.f. $P(X)$.

- **Expected Value** of X:

$$\mu_x \equiv E(X) = \sum_{i=1}^{n} x_i \cdot P(X = x_i)$$

- **Variance** of X:

$$\sigma_x^2 \equiv Var(X) = \sum_{i=1}^{n} \left(x_i - \mu_x\right)^2 \cdot P(X = x_i)$$

- **Standard Deviation** of X: σ_x

Example

- Consider an experiment of tossing 2 coins, and let $X = \#$ of Heads.
- Compute μ_x and σ_x^2?
- First find the p.d.f. of X

	Number of Heads		
	0	1	2
$P(X=x)$	0.25	0.50	0.25

- Then compute

$$\mu_x = \sum_{i=1}^{3} x_i \cdot P(X = x_i) = 0 \cdot (0.25) + 1 \cdot (0.50) + 2 \cdot (0.25) = 1$$

$$\begin{aligned}\sigma_x^2 =& \sum_{i=1}^{3} \left(x_i - \mu_x\right)^2 \cdot P(X = x_i) \\ =& (0-1)^2 \cdot (0.25) + (1-1)^2 \cdot (0.50) + (2-1)^2 \cdot (0.25) = 0.5\end{aligned}$$

Linear Functions of A Random Variable

- Suppose X is a discrete random variable with some pdf $P(X)$.
- In this section, we will see how we can generate new r.v.s from X and how the expected value and variance of these new r.v.'s are related to μ_x and σ_x^2.
- Let us begin with the following

$$Y = a + bX$$

  where a and b are constants.
- **Fact:** Y is also a r.v. and $P(Y)$ is completely determined by $P(X)$

Example

- Suppose our original r.v. X has the following probability distribution

X	-2	1	2
P(X)	.5	.3	.2

- And let us define a new random variable based on X as

$$Y = 200 + 30X$$

- Note that

$$X = -2 \iff Y = 140$$
$$X = 1 \iff Y = 230$$
$$X = 2 \iff Y = 260$$

- Therefore, we obtain $P(Y)$ as

Y	140	230	260
P(Y)	.5	.3	.2

Expected Value of a Linear Combination of a R.V.

- Whenever we have a random variable we can calculate its expected value and variance, no matter how complicated the r.v. is.
- Let us compute the expected value for each of the r.v.s X and Y

$$E(X) = \sum_{x \in \{-2,1,2\}} xP(X = x) = (-2)(.5) + (1)(.3) + (2)(.2) = -0.3$$

$$E(Y) = \sum_{y \in \{140,230,260\}} yP(Y = y) = 140(.5) + 230(.3) + 260(.2) = 191$$

- Now notice the relationship between $E(X)$ and $E(Y)$:

$$E(Y) = 200 + 30E(X) = 200 + 30(-0.3) = 191$$

- **Result 1** Let X be r.v. with $\mu_x = E(X)$, then for $Y = a + bX$:

$$E(Y) = a + bE(X) \quad \text{or} \quad \mu_y = a + b\mu_x$$

Variance of a Linear Combination of a R.V.

- Now, let us calculate the variances for each of the r.v.s

$$\sigma_X^2 = \sum_{x \in \{-2,1,2\}} (x - \mu_X)^2 P(X = x)$$
$$= (-2 + 0.3)^2(.5) + (1 + 0.3)^2(0.3) + (2 + 0.3)^2(0.2) = 3.01$$
$$\sigma_Y^2 = \sum_{y \in \{140,230,260\}} (y - \mu_Y)^2 P(Y = y)$$
$$= (140 - 191)^2(.5) + (230 - 191)^2(0.3) + (260 - 191)^2(0.2) = 2709$$

- Now notice the relationship between σ_X^2 and σ_Y^2:

$$\sigma_Y^2 = 30^2 \sigma_X^2 = 900(3.01) = 2709$$

- **Result 2** Let X be r.v. with $Var(X) = \sigma_x^2$, then for $Y = a + bX$

$$Var(Y) = b^2 Var(X) \quad \text{or} \quad \sigma_y^2 = b^2 \sigma_x^2 \quad \text{and also} \quad \sigma_y = |b|\sigma_x$$

Jointly Distributed Discrete Random Variables

Sometimes it is more useful and/or natural to study two random variables simultaneously. Consider the following example:

- Suppose you are part of a team studying the relationship between smoking habits and cancer.
- As a part of the study you surveyed a randomly chosen 1000 males between ages 60-65 across the country.
- In the survey, first you ask whether they smoke and if they smoke, whether they are light or heavy smokers. Finally, you ask if they have been diagnosed with any type of cancer.
- Each observation will fall into only one of the cells in the table

$Smoking/Cancer$	Yes	No
Non-smoker	·	·
Light-smoker	·	·
Heavy-smoker	·	·

- Suppose your survey have produced the following data

$Smoking/Cancer$	Yes	No
Non-smoker	20	580
Light-smoker	35	215
Heavy-smoker	45	105

- How can we think of this problem in terms of random variables?
- To do this let us define the following two random variables
  - C= Cancer history; $C = 0$ for Yes, $C = 1$ for No
  - S= Smoking habit; $S = 0$ for non-smokers, $S = 1$ for light-smokers, $S = 2$ for heavy-smokers
- Now we can represent the same table in terms of the r.v.'s C and S

$S(row)/C$	0	1
0	.020	.580
1	.035	.215
2	.045	.105

$S(row)/C$	0	1
0	.020	.580
1	.035	.215
2	.045	.105

- For example, $P(S = 1, C = 0) = 0.035$ is the probability that a randomly chosen male in that age group is a light-smoker **and** diagnosed with cancer
- Probability of randomly selected male to be a non-smoker?

$$P(S = 0) = P(S = 0, C = 0) + P(S = 0, C = 1) = 0.020 + 0.580 = 0.6$$

- Similarly, we can compute $P(S = 1) = 0.25$ and $P(S = 2) = 0.15$, and then construct **marginal probability distribution** table for S as

S	0	1	2
P(S=s)	.60	.25	.15

- Since we have already obtained $P(S)$, $E(S)$ and $V(S)$ can be easily computed as usual. [Verify that $E(S) = .55$ and $V(S) = .366$.]

$S(row)/C$	0	1
0	.020	.580
1	.035	.215
2	.045	.105

- With this data what else we can do? (Cont'd)
  - Probability of a randomly selected male having experienced cancer given that the person is a non-smoker?

$$P(C = 0|S = 0) = \frac{P(S = 0, C = 0)}{P(S = 0)} = \frac{0.02}{0.60} = 0.033$$

    Similarly, we can also calculate $P(C = 1|S = 0)$. These two are called **conditional probabilities** of C given the person is a non-smoker

- The most important thing to observe is that $C|(S = 0)$ is a r.v. because although we know that $S = 0$, we do not know whether $C = 0$ or $C = 1$: each of these two possibilities have their respective probabilities, denoted by $P(C = 0|S = 0)$ and $P(C = 1|S = 0)$, respectively.

- If you wish you can give another name to $C|S=0$. For example, say you defined $Z=(C|S=0)$. Then Z is either 0 or 1, and we can calculate

$$P(Z=0)=P(C=0|S=0) \text{ and } P(Z=1)=P(C=1|S=0)$$

- Since $C|(S=0)$ is a r.v. we should be able to calculate its expected value and variance, right?
- Let us summarize what we have done so far under the probability distribution of $C|S=0$ as follows (in two alternative ways)

$C\|(S=0)$	0	1
$P(C\|S=0)$	.03	.97

Z	0	1
$P(Z)$	.03	.97

- Now compute its expected value and variance:

$$E(C|S=0)=\mu_{C|S}=\sum_{c\in\{0,1\}} cP(C=c|S=0)$$
$$=0\times(.03)+1\times(.97)=0.97$$
$$Var(C|S=0)=\sum_{c\in\{0,1\}}(\mu_{C|S}-c)^2P(C=c|S=0)$$
$$=(0-.97)^2(.03)+(1-.97)^2(0.97)=0.0291$$

Definitions

- A **joint probability distribution** is used to express the probability that simultaneously X takes on the specific value x and Y takes on the value y:

$$P(x, y) = P(X = x, Y = y)$$

- The **marginal probability distributions** are

$$P(X = x) = \sum_y P(x, y), \qquad P(Y = y) = \sum_x P(x, y)$$

- The **conditional probability distribution** of the random variable Y expresses the probability that Y takes the value y when the value x is specified for X:

$$P(y|x) = \frac{P(x, y)}{P(x)}$$

- The jointly distributed random variables X and Y are said to be **independent** if the following condition is satisfied

$$P(x, y) = P(x)P(y)$$

  for all possible (x, y) pairs.

Definitions Cont'd

- The **conditional mean** of X given $Y = y^*$ is

$$\mu_{X|Y} = E[X|Y] = \sum_x x \cdot P(X = x|Y = y^*)$$

Notation: In order to clarify on which particular value of Y conditioned on we could have used the notation $\mu_{X|Y=y^*} = E[X|Y = y^*]$. But as long as it is clear from the content that it is conditioned on $Y = y^*$ we use the notation in the above equation. If you would like to simplify the notation further, you can denote $P(X = x|Y = y^*)$ by $P(x|y^*)$.

- The **conditional variance** of X given $Y = y^*$ is

$$\sigma^2_{X|Y} = \sum_x (x - \mu_{X|Y})^2 P(X = x|Y = y^*)$$

- If you define a new r.v. $Z = (X|Y = y^*)$ and find its pdf, the conditional variance defined above equals

$$\sigma^2_Z = \sum_z (z - \mu_z)^2 P(z)$$

Covariance

- Let X and Y be two discrete r.v.s with means μ_x and μ_y, respectively
- The expected value of $(X - \mu_x)(Y - \mu_y)$ is called **covariance** between X and Y:

$$\begin{aligned} Cov(X,Y) &= E\big[(X - \mu_x)(Y - \mu_y)\big] \\ &= \sum_x \sum_y (x - \mu_x)(y - \mu_y)P(x,y) \end{aligned}$$

- Sometimes the following might be easier to compute

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

- If two random variables are statistically independent, the covariance between them is 0. However, the converse is not necessarily true.

Covariance: Example

- As an example, let us compute the covariance between smoking habits and cancer, i.e. $Cov(S,C) = ?$

$$
\begin{aligned}
Cov(S,C) &= E\big[(S-\mu_S)(C-\mu_C)\big] \\
&= \sum_s \sum_c (s-\mu_S)(c-\mu_C)P(s,c) \\
&= (0-.55)(0-.9)(.02) \\
&+ (0-.55)(1-.9)(.580) + \dots \\
&+ \dots (2-.55)(1-.9)(.105) = ?
\end{aligned}
$$

S/C	0	1
0	.020	.580
1	.035	.215
2	.045	.105

$\mu_S = .55$ and $\mu_C = .9$

Correlation

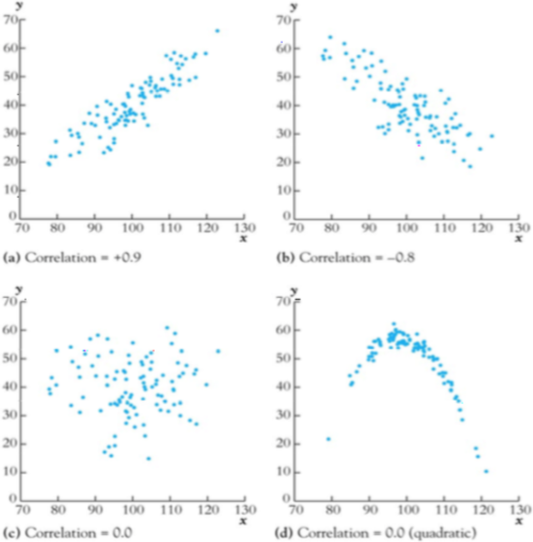
- Covariance is unit sensitive and only tells the direction of the relationship, not the strength of it.
- **Correlation Coefficient** measures the direction and relative strength of the linear relationship between two r.v.s

$$\rho = Corr(X, Y) = \frac{Cov(X, Y)}{\sigma_x \sigma_Y}$$

Properties and interpretation of ρ

- $-1 \leq \rho \leq 1$
- $\rho = 0$: no linear relationship between X and Y
- $\rho > 0$: positive linear relationship between X and Y, i.e. when X is high (low) then Y is likely to be high (low).
- $\rho < 0$: negative linear relationship between X and Y, i.e. when X is high (low) then Y is likely to be low (high)
- $\rho = 1$ $(\rho = -1)$ there is a perfect positive (negative) linear dependency between X and Y.

Correlation: Examples



(a) Correlation = +0.9

(b) Correlation = −0.8

(c) Correlation = 0.0

(d) Correlation = 0.0 (quadratic)

Linear Functions of Two Random Variables

- Just like in the single r.v. case, when we have two r.v.s X and Y we can generate new r.v.s: $W_1 = 3X + Y$, $W_2 = X^2 - 5Y$, etc.
- In this course, we are going to be interested only in linear combinations of two r.v.s:

$$W = aX \pm bY$$

- The **expected value** of the linear combination of two r.v.s

$$E[W] = aE(X) \pm bE(Y)$$
$$\text{or } \mu_w = a\mu_x \pm b\mu_y$$

- The **variance** for the linear combination of two r.v.s

$$Var[W] = a^2Var(X) + b^2Var(Y) \pm 2abCov(X,Y)$$
$$\text{or } \sigma_w^2 = a^2\sigma_x^2 + b^2\sigma_y^2 \pm 2abCov(X,Y)$$

- As an example, let us look at the following portfolio example.

Example: Portfolio Analysis

- Let
  - r.v. X be the price of stock A
  - r.v. Y be the price of stock B
- Then the market value, W, for the portfolio is given by the linear function

$$W = aX + bY$$

  where a and b are the # of shares of stock A and B, respectively.
- The expected market value of the portfolio:

$$E[W] = aE(X) + bE(Y)$$

- The variance of the market value of the portfolio:

$$Var[W] = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$$

- How would you interpret the term $2abCov(X, Y)$ above?

Example: Investment Returns

Consider a portfolio manager's problem: there are two investment alternatives and returns from each depends on the state of the economy:

		Investment Options	
$P(x_i, y_i)$	Economic Cond.	Passive Fund X	Active Fund Y
0.2	Recession	-\$25	-\$200
0.5	Stable Econ.	+\$50	+\$60
0.3	Expanding Econ.	+\$100	+\$350

Table: Return per \$1000 for two types of investments

- Interpret the table?
- Represent this table as the joint distribution of X and Y?

- The expected value of each fund:

$$\mu_x = E(X) = (-25)(.2) + (50)(.5) + (100)(.3) = 50$$
$$\mu_y = E(Y) = (-200)(.2) + (60)(.5) + (350)(.3) = 95$$

- The variance of each fund:

$$\sigma_x^2 = (-25-50)^2(.2) + (50-50)^2(.5) + (100-50)^2(.3) = (43.3)^2$$
$$\sigma_y^2 = (-200-95)^2(.2) + (60-95)^2(.5) + (350-95)^2(.3) = (193.71)^2$$

- What is the trade-off between investing in X versus Y?
- The covariance between X and Y:

$$\begin{aligned} Cov(X,Y) &= (-25-50)(-200-95)(.2) + (50-50)(60-95)(.5) \\ &\quad + (100-50)(350-95)(.3) \\ &= 8250 \end{aligned}$$

Suppose %40 of the portfolio (P) is in investment X and %60 is in investment Y.

- Define P as a linear combination of X and Y

$$P = 0.4X + 0.6Y$$

- Find the expected value and the variance of P

$$\mu_p = (.4)(50) + (.6)(95) = 77$$
$$\sigma_p^2 = (.4)^2(43.3)^2 + (.6)^2(193.21)^2 + 2(.4)(.6)(8250) = (133.04)^2$$

- The portfolio return and portfolio variability are between the values for investments X and Y considered individually.
- The aggressive fund has a higher expected return, but much more risky as well
- The covariance of 8250 indicates that the two investments are positively related and will vary in the same direction. Is that good in terms of risk diversification?

Review Example: Tennis vs Golf

Suppose that X is the number of tennis rackets and Y is the number of golf clubs sold daily in a small sports store, and their joint probability distribution is as in the table below

$X(row)/Y(col.)$	1	2	3
1	.30	.18	.12
2	.15	.09	.06
3	.05	.03	.02

1. Determine the marginal probability distributions of X and Y.

X	1	2	3
P(X)	.6	.3	.1

Y	1	2	3
P(Y)	.5	.3	.2

2. Are X and Y independent? Explain.

   X and Y are independent since $P(x, y) = P(x) \cdot P(y)$ for all pairs (x, y). For example,

$$P(X = 1, Y = 1) = P(X = 1) \cdot P(Y = 1)$$
$$0.3 = (0.6) \cdot (0.5)$$

   Verify that this condition is satisfied for all the other pairs (x, y).

$X(row)/Y(col.)$	1	2	3
1	.30	.18	.12
2	.15	.09	.06
3	.05	.03	.02

3. Calculate the conditional probability $P(Y = 3|X = 1)$.

$$P(Y = 3|X = 1) = \frac{P(X=1 \text{ and } Y=3)}{P(X=1)} = \frac{0.12}{0.60} = 0.20$$

4. Calculate the expected values of X and Y, i.e, E(X) and E(Y).

To calculate E(X) and E(Y) we need the marginal probabilities from part (a).

$$\mu_x = \sum_{x \in \{1,2,3\}} x \cdot P(x) = 1.(0.6) + 2.(0.3) + 3.(0.1) = 1.5$$

$$\mu_y = \sum_{y \in \{1,2,3\}} y \cdot P(y) = 1.(0.5) + 2.(0.3) + 3.(0.2) = 1.7$$

5. Calculate the variance for X and Y.

$$\sigma_y^2 = \sum_{x \in \{1,2,3\}} (x - \mu_x)^2 P(X = x)$$
$$= (1 - 1.7)^2(0..5) + (2 - 1.7)^2(0.3) + (3 - 1.7)^2(0.2) = 0.61$$

Similarly, verify that $\sigma_x^2 = .45$.

6. Calculate $E(XY)$.

We can think of in two different ways:

(a) Define $Z = XY$, construct the pdf for Z, then calculate $E(Z)$

Z	1	2	3	4	6	9
P(z)	.30	.33	0.17	.09	.09	.02

(Note that we get $Z = 2$ for two different (x, y) pairs, $(1, 2)$ and $(2, 1)$, so we need to add them up to get $P(Z = 2)$. In another words, $P(Z = 2) = P(1, 2) + P(2, 1) = 0.18 + 0.15 = 0.33$. We did the same thing to compute $P(Z = 3)$ and $P(Z = 6)$. Check!)

$$E(Z) = \sum_z z \cdot P(z) = 1(.3) + 2(.33) + \ldots + 9(.02) = 2.55$$

(b) For each (x, y) pair compute $x \cdot y \cdot P(x, y)$, and add them up:

$$E(XY) = (1)(1)(.3) + (1)(2)(.18) + \ldots + (3)(3)(.02) = 2.55$$

7. Calculate Cov(X,Y). Did you expect this answer? Why?

$Cov(X,Y) = E(XY) - E(X)E(Y) = 2.55 - (1.70)(1.50) = 0.$
Yes, this was expected since X and Y are independent, from part (c).

8. Find the pdf of the r.v. $W = X + Y$

W	2	3	4	5	6
P(w)	.3	.33	.26	.09	.02

9. Compute $E(W)$ and $Var(W)$ directly by using the pdf of W

$$\begin{aligned}\mu_w =& \sum_w w \cdot P(W = w)\\ =& 2(.3) + \ldots + 6(.02)\\ =& 3.2\end{aligned}$$

$$\begin{aligned}\sigma_w^2 =& \sum_w (w - \mu_w)^2 \cdot P(W = w)\\ =& (2 - 3.2)^2(.3) + \ldots + (6 - 3.2)^2(.02)\\ =& 1.06\end{aligned}$$

10. Compute $E(W)$ and $Var(W)$ by using the formulas given for the r.v.s that are linear combinations of two r.v.s

$W = aX + bY$, so here $a = 1$ and $b = 1$. Therefore,

$$\begin{aligned}\mu_w =& 1.\mu_x + 1.\mu_y\\ =& 1.5 + 1.7\\ =& 3.2\end{aligned}$$

$$\begin{aligned}\sigma_w^2 =& (1)^2\sigma_x^2 + (1)^2\sigma_y^2 + 2(1)(1)Cov(X,Y)\\ =& 0.61 + 0.45 + 2 \cdot 0\\ =& 1.06\end{aligned}$$

11. Show that $Var(X+Y) = Var(X) + Var(Y)$. Did you expect this result? Why?

$Var(X) + Var(Y) = .61 + .45 = 1.06$ which is equal to $Var(W = X + Y) = 1.06$. Yes, since X and Y are independent random variables.

12. Compute $P(X = 2 \mid X + Y = 4)$=?

Since we know that $X + Y = 4$, we can forget all (x,y) pairs that doesn't satisfy this condition. So the joint probability distribution reduces to the following

$X(row)/Y(col.)$	1	2	3
1	.30	.18	**.12**
2	.15	**.09**	.06
3	**.05**	.03	.02

Then,

$$P(X = 2 \mid X + Y = 4) = \frac{.09}{.12 + .09 + .05} = 9/26$$

Review Example: A Portfolio Problem

Suppose you are asked to analyze an investment portfolio that contains 5 units of shares stock A and 10 shares of stock B. The joint probability distribution of the stock prices is presented in the table below.

	Stock B			
Stock A	\$35	\$55	\$80	$P(A)$
\$50	.10	?	?	.30
\$60	.10	.20	?	.40
\$70	?	.15	.10	?
$P(B)$	?	.50	.25	1.00

1. Interpret the table.
2. Complete the missing joint and marginal probabilities in the table.
3. Compute the expected value of the portfolio.
4. Compute the standard deviation of the portfolio.
5. What is the expected price of Stock A given that Stock B is \$55?

1. For example, $P(A = \$60, B = \$55) = 0.20$, and so on. Also, numbers at the end of each row and column denote the marginal probabilities, i.e., $P(B = \$55) = 0.50$, etc.
2. Completed table

	Stock B			
Stock A	\$35	\$55	\$80	$P(A)$
\$50	.10	.15	.05	.30
\$60	.10	.20	.10	.40
\$70	.05	.15	.10	.30
$P(B)$	.25	.50	.25	1.00

3. We represent this portfolio as

$$W = 5A + 10B$$

To compute μ_w:

$$\begin{aligned} \mu_A &= 50(.3) + 60(.4) + 70(.3) = 60 \\ \mu_B &= 35(.25) + 55(.5) + 80(.25) = 56.3 \\ \Longrightarrow \mu_w &= 5(60) + 10(56.3) = 862.5 \end{aligned}$$

4. First compute

$$
\begin{aligned}
\sigma_A^2 &= (50-60)^2.3 + (60-60)^2.4 + (70-60)^2.3 = 60 \\
\sigma_B^2 &= 254.7 \\
Cov(A,B) &= (50-60)(35-56.3)(.10) + (60-60)(35-56.3)(.10) \\
&\quad + (70-60)(35-56.3)(.05) + \ldots + (70-60)(80-56.3)(.10) \\
&= 22.5
\end{aligned}
$$

Now we can compute σ_w^2:

$$
\begin{aligned}
\sigma_w^2 &= 5^2\sigma_B + 10^2\sigma_B + 2(5)(10)Cov(A,B) \\
&= 5^2(60) + 10^2(254.69) + 2(5)(10)(22.5) \\
&= 29,219 \\
\sigma_w &= \sqrt{29,219} = 170.9
\end{aligned}
$$

5. Let us first construct the probability distribution table for the random variable $A|B = 55$:

$A\|B = 55$	50	60	70
$P(A\|B = 55)$	.15/.50	.20/.50	.15/.50

Then, we can compute the expected value of this r.v. as usual:

$$E(A|B = 55) = 50\left(\frac{0.15}{0.50}\right) + 60\left(\frac{0.20}{0.50}\right) + 70\left(\frac{0.15}{0.50}\right) = 60$$