Midterm II

Applied Statistics and Econometrics II
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1. Let $X \sim \text{Poisson}(\theta)$ with the p.m.f.

$$P(X = x) = \frac{\theta^x e^{-\theta}}{x!}$$

   (a) Find the mle of θ.

   (b) Let the table represent a summary of a random sample of size 60 from the Poisson

x	0	1	2	3	4	5	6	7
Frequency	7	14	12	13	6	3	3	2

   distribution above. Find the maximum likelihood estimate of $P(X = 3)$.

2. Let $\{X_1, \ldots, X_5\}$ be an iid random sample from a uniform distribution with the support $(0, \theta)$. In this problem, you are asked to construct the likelihood ratio test to test the null hypothesis that the true parameter value is $\theta_0 = 10$. As data suppose we have a random sample of size $n = 5$ at hand with the maximum value of 8.

   (a) State the null and alternative hypotheses.

   (b) Calculate the likelihood ratio test statistic

$$\Lambda = \frac{L(\theta_0)}{L(\hat{\theta})}$$

   where $\hat{\theta}$ is the mle of θ.

   (c) Conclude the test using the following decision rule,

$$\text{Reject } H_0 \text{ in favor of } H_1 \text{ if } \Lambda \leq c,$$

   where c is such that $\alpha = P_{\theta_0}[\Lambda \leq c]$ and take $\alpha = 0.1$.

3. A researcher uses a sample of 200 quarterly observations on Y_t, the number (in 1000s) of unemployed people, to model the time series behaviour of the series and to generate predictions. First, he computes the sample autocorrelation and the sample partial autocorrelation functions, respectively, with the following results:

k	1	2	3	4	5	6	7	8
$\hat{\rho}_k$	0.83	0.71	0.60	0.45	0.44	0.35	0.29	0.20
$\hat{\theta}_{kk}$	0.83	0.16	-0.09	0.05	0.04	-0.05	0.01	0.01

(a) What do we mean by the sample autocorrelation and the sample partial autocorrelation functions? Why is the first partial autocorrelation equal to the first autocorrelation coefficient (0.83)

(b) Does the above pattern indicate that an autoregressive or a moving average representation is more appropriate? Why?

Suppose the researcher decides to estimate, as a first attempt, an AR(1) model

$$Y_t = \delta + \theta Y_{t-1} + \varepsilon_t$$

and the OLS produces an estimated value of 0.83 for θ with a standard error of 0.07.

(c) Can OLS produce a consistent estimate of θ in this set up? Explain breifly why.

(d) The researcher wants to test for a unit root. What is meant by 'a unit root'? What are the implications of the presence of a unit root? Why are we interested in it? (Give statistical or economic reasons.)

(e) Formulate the hypothesis of a unit root and perform a unit root test based on the above regression.

Next, the researcher extends the model to an AR(2), with the following results (standard errors in parentheses):

$$\underset{}{Y_t} = \underset{(5.67)}{50} + \underset{(0.07)}{0.74}\, Y_{t-1} + \underset{(0.07)}{0.16}\, Y_{t-2} + e_t$$

(f) Would you prefer the AR(2) model to the AR(1) model? How would you check whether an ARMA(2, 1) model may be more appropriate?

(g) How would you test for the presence of a unit root in the AR(2) model?

(h) From the above estimates, compute an estimate for the average number of unemployed, $E[Y_t]$.

(i) Suppose the last two quarterly unemployment levels for 2017:III and 2017:IV were 550 and 600, respectively. Compute forecasts for 2018:I and 2018:II.

4. Consider the process

$$Y_t = \delta + \phi Y_{t-1} + w_t,$$

where w_t is a white noise process with variance σ_w^2 and let $|\phi| < 1$ be a constant.

(a) Show that

$$Y_t = \frac{\delta}{1-\phi} + \sum_{j=0}^{t} \phi^j w_{t-j}$$

(b) Find $E[Y_t]$.

(c) Show that,

$$\text{Var}[Y_t] = \frac{\sigma_w^2}{1-\phi^2}\left(1 - \phi^{2(t+1)}\right), \qquad t = 0, 1, \ldots$$

(d) Show that, for $h \geq 0$,

$$\text{Cov}(Y_t, Y_{t+h}) = \phi^h \text{Var}(Y_t)$$

(e) Is Y_t stationary?

(f) Argue that, as $t \to \infty$, the process becomes stationary, so in a sense, Y_t is "asymptotically stationary".

	Without trend		With trend	
Sample size	1 %	5 %	1 %	5 %
$T = 25$	−3.75	−3.00	−4.38	−3.60
$T = 50$	−3.58	−2.93	−4.15	−3.50
$T = 100$	−3.51	−2.89	−4.04	−3.45
$T = 250$	−3.46	−2.88	−3.99	−3.43
$T = 500$	−3.44	−2.87	−3.98	−3.42
$T = \infty$	−3.43	−2.86	−3.96	−3.41

Figure 1: 1% and 5% critical values for Dickey-Fuller tests