Practice Problems 1

Econometrics, Spring 2019
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[1] Suppose that Y_i takes on 0 and 1 (a Bernoulli random variable) with the probability distribution

$$P[Y_i = 0] = 0.22, \quad P[Y_i = 1] = 0.78, \quad \text{for all } i = 1, 2, 3, \ldots$$

a) Find $E[Y_1]$ and $Var[Y_1]$.
b) Find the sampling distribution of $\overline{Y}$ (sample mean) for a sample size of 3.
c) Plot the distribution of $\overline{Y}$.
d) Does the distribution of $\overline{Y}$ look like a normal distribution? Discuss whether the Central Limit Theorem applies to this example.

[2] Suppose that Y_i takes on 0, 1 and 2 with the probability distribution

$$P[Y_i = 0] = 0.2, \quad P[Y_i = 1] = 0.30, \quad P[Y_i = 2] = 0.30 \text{ for all draws } i = 1, 2, 3, \ldots$$

a) Find $E[Y_1]$ and $Var[Y_1]$.
b) Find the sampling distribution of $\overline{Y}$ (sample mean) for a sample size of 2.
c) Plot the distribution of $\overline{Y}$.
d) Does the distribution of $\overline{Y}$ look like a normal distribution? Discuss whether the Central Limit Theorem applies to this example.

[3] Use the probability distribution given in the table below to compute

(Table 2.2)	Rain (X=0)	No Rain (X=1)	**Total**
Long commute (Y=0)	0.15	0.07	0.22
Short commute (Y=1)	0.15	0.63	0.78
Total	0.30	0.70	1.00

a) $E(Y)$ and $E(X)$
b) σ_Y^2 and σ_X^2
c) σ_{XY} and corr(X, Y)

[4] **[SW 2.3]** Using the random variables X and Y from Table 2.2, consider two new random variables $W = 3 + 6X$ and $V = 20 - 7Y$. Compute

a) $E(W)$ and $E(V)$
b) σ_W^2 and σ_V^2
c) σ_{WV} and corr(W, V)

[5] **[SW 2.6]** The following table gives the joint probability distribution between employment status and college graduation among those either employed or looking for work (unemployed) in the working-age U.S. population for 2012.

	Employment Status		
	Unemployed (Y=0)	Employed (Y=1)	**Total**
Non-college grads (X=0)	0.053	0.586	0.639
College grads (X=1)	0.015	0.346	0.361
Total	0.068	0.932	1.00

a) Compute $E(Y)$.

b) The unemployment rate is a fraction of the labor force that is unemployed. Show that the unemployment rate is given by $1 - E(Y)$.

c) Calculate $E(Y|X = 1)$ and $E(Y|X = 0)$.

d) Calculate the unemployment rate for (i) college graduates and (ii) non-graduates.

e) A randomly selected member of this population reports being unemployed. What is the probability that this worker is a (i) college graduate, (ii) non-graduate?

f) Are educational achievement and employment status independent? Explain.

[6] [**SW 2.7**] In a given population of two-earning male-female couples, male earnings have a mean of $40,000 per year and a standard deviation of $12,000. Female earnings have a mean of $45,000 per year and a standard deviation of $18,000. The correlation between male and female earnings for a couple is 0.80. Let C denote the combined earnings for a randomly selected couple.

a) What is the mean of C?

b) What is the covariance between male and female earnings?.

c) What is the standard deviation of C?

d) Convert the answers to (a) through (c) from U.S. dollars to euros (assume that the exchange rate between the two is 0.86).

[7] Compute the following probabilities:

a) If Y is distributed $N(3, 9)$, find $P(Y > 0)$.

b) If Y is distributed $N(50, 25)$, find $P(40 \leq Y \leq 52)$.

c) If Y is distributed t_{15}, find $P(Y > 1.75)$.

d) If Y is distributed t_{90}, find $P(-1.99 \leq Y \leq 1.99)$.

e) If Y is distributed $N(0, 1)$, find $P(-1.99 \leq Y \leq 1.99)$.

f) Why are the answers to (d) and (e) are approximately the same?

[8] The random variable Y has a mean of 12 and a variance of 4.5 and let $Z = \frac{1}{3}Y + 5$.

a) Do you need any information regarding the distribution of Y to find the mean and the variance of Z? If not, find $E(Y)$ and $V(Y)$.

b) Can you compute $P(8.5 \leq Z \leq 10)$? What if it was given that, in addition, Y is distributed normally: $Y \sim N(12, 3)$. If yes, compute that probability.

[9] [**SW 2.14**] In a population $\mu_Y = 100$ and $\sigma_Y^2 = 43$. Use the Central Limit Theorem to answer the following questions:

a) In a random sample of size $n = 100$, find $P(\overline{Y} \leq 101)$.

b) In a random sample of size $n = 165$, find $P(\overline{Y} > 98)$.

c) In a random sample of size $n = 64$, find $P(101 \leq \overline{Y} \leq 103)$.

[10] Y_i, $i = 1, \ldots, n$ are i.i.d. Bernoulli random variables with $p = 0.4$ (i.e. $P(Y_i = 1) = 0.4$ and $P(Y_i = 0) = 0.6$). Let $\overline{Y}$ denote the sample mean.

a) Use the Central Limit Theorem to compute approximations for

i. $P(\overline{Y} \geq 0.43)$ when $n = 100$.

ii. $P(\overline{Y} \leq 0.37)$ when $n = 400$.

b) How large would n need to be to ensure that $P(0.39 \leq \overline{Y} \leq 0.41) \geq 0.95$? (Hint: Use the central limit theorem to compute an approximate answer.)

[11] In any year, the weather may cause damages to a home. On a year-to-year basis, the damage is random. Let Y denote the dollar value of damages in any given year. Suppose that during 95% of the year $Y = \$0$, but during the other 5% $Y = 20,000$.

a) What are the mean and standard deviation of damages caused in a year?

b) Consider an "insurance pool" of 100 people whose homes are sufficiently dispersed so that, in any year, the damage to different homes can be viewed as independently distributed random variables. Let $\overline{Y}$ denote the average damage caused to these 100 homes in one year.

i. What is the expected value of the average damage $\overline{Y}$?

ii. What is the probability that $\overline{Y}$ exceeds \$2,000? (Hint: Use the central limit theorem to compute an approximate answer.)

[12] The following table displays the joint probability distribution of two discrete random variables X and Y:

	X=1	X=2	X=3
Y=0	.10	.12	.06
Y=1	.05	.10	.11
Y=2	.02	.16	.28

a) Determine the marginal probability distribution of Y.

b) Compute the expected value and the standard deviation of Y?

c) Compute the expected value of Y given that $X = 3$, i.e. $E(Y|X = 3) =$?

d) For $T = Y^3 + 2Y$, compute the expected value of T, i.e. $E(T) =$?

[13] Use the data given in the table below to compute

	Agree	Neutral	Disagree
Correspondence	3	2	1
# of Respondents	8	4	2

a) Mean

b) Standard Deviation

c) Change correspondence to 2, 1, 0. Now redo the previous parts.