The Social Value of *Credible* Public Information*

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Abstract

In this paper, I study the model in Morris and Shin (2002) under *strategic information transmission constraints* between the public authority and private agents in order to investigate how the precision of the public signal affect the quality of the communication. In the model, a central bank (CB) communicates with two private agents. All three players have different interests over the state of the economy and receive informative but noisy signals when the state is realized. I find that CB only cares about the average bias of the private agents, and CB can truthfully reveal her signal only if the average bias is below a certain threshold. More importantly, as the CB's signal precision increases, the possibility of truthful communication increases in her signal precision which in return enhances her ability to communicate with the private agents.

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1 Introduction

In a very influential paper Morris and Shin (2002) study a model in which a continuum of agents receive both public and private signals on the underlying fundamentals, and aim to take actions to maximize their utility functions which depend on the state of those of fundamentals as well as some individual parameters. In the model agents engage in a zero-sum race to second-guess the actions of other individuals in which a player's prize depends on the distance between his own action and the average of actions of others. They conclude that the welfare effect of better public information provision is ambiguous. Specifically, the greater the precision of the agents' private information, the more likely the increased provision of the public information lowers the social welfare. The detrimental effect of the public information arises from the fact that the coordination motive entails placing too much weight on the public signal relative to the weight that would be used by a social planner, which in turn implies that any noise in the public signal may lead people to coordinate on a suboptimal point.

Following Morris and Shin (2002) there have been many studies approaching the problem from different points of views[1]. In all these papers, released public information is assumed to be perfectly credible, and therefore, to incorporate public and private information into their decisions, agents simply use a Bayesian updating formula. However, the main lesson that we know from the cheap talk literature is that *perfect communication is not to be expected in general unless agents' interests are completely alligned*. In a type of model that I just outlined, it is hard to ignore the fact that the information released by the public authorities (say, by the central bank) might be perceived as strategic information by the private agents (investors) as they might have different interests over the state of the economy. Therefore, the public information does not need to be perceived as fully credible.

This observation motivated me to study a Morris and Shin (2002) type of model with strategic information transmission constraints among the two sides in order investigate how the precision of the signals effects the quality of the communication

[1] See Veldkamp(2011, Veldkamp (2011)) chapters 4-5 for a recent survey of this literature

and the social welfare. In the next section I present a simple model along these lines.

2 Model

There is a central bank (CB) and two players ($i = 1, 2$) with the following utility functions:

$$U_{CB} = -(a_1 - \theta)^2 - (a_2 - \theta)^2 \quad (1)$$

and

$$U_i = -(1-r)(a_i - \theta - b_i)^2 - r(a_1 - a_2)^2 , \qquad i = 1, 2 , \quad (2)$$

where θ is an unknown state variable with the distribution function $F(\theta)$, r is the degree of complementarity with $0 < r < 1$, and b_i's are individual biases with $b_1 > b_2 > 0$.

Timing in the model is as follows:

1. Nature draws the state θ.

2. CB and the players receive informative signals y and x_i, respectively, in the following form

$$y = \theta + \eta$$

and

$$x_i = \theta + \epsilon_i$$

where η and ϵ_i are normally distributed with mean zero and variances σ_η^2 and σ_ϵ^2, respectively. I also assume that public and private noises are independent of each other and of θ, and private noises are not correlated across individuals, $E(\epsilon_i \epsilon_j) = 0$ for $i \neq j$.

3. CB sends a public signal $s(y)$ to the players.

4. Upon receiving signals the players take actions conditional on their information sets $\{s(y), x_i\}$.

Note that here CB does not need to share all the information she has, instead she sends a signal to maximize the social welfare as represented by the utility function in equation (1). As the players know that CB maximizes the social welfare, they do not treat $s(y)$ as a fully credible piece of information. This description of the game is a cheap talk game (Crawford and Sobel (1982)) with two audiences and complementarities: the sender (CB) imperfectly observes the state, and receivers also have some private imperfect information in addition to the sender's message.

Although, I use central bank analogy in the model, alternatively the side making public announcements could be thought of as a party leader communicating to partisans. If their interests are not perfectly aligned, then all the information coming from the leader might be considered as strategic. The only important thing here is that only the side making the public announcements has access to some private communication means with the other side, and all three players' preferences are not perfectly aligned.

In the next section I summarize some of the related literature before starting to study the model.

3 Related Literature

The most related studies to my model are Chen (2009) and Goltsman and Pavlov (2011). Chen studies both one-way communication in which only an expert reports and two-way communication in which the decision maker communicates first before the expert reports. She finds that in the one-way communication game non-monotone equilibria may arise, and in the two-way communication case she characterize the conditions under which truthful communication by the decision maker fails in equilibrium. Goltsman and Pavlov discuss how the sender should design the communication with the receiver when both public and private messages are at his disposal. Other than the question raised being totally different, my paper differs from Chen's by dealing with multiple agents, and from Golstman and Pavlov's model by extending their model to a two-sided incomplete information set up. Moreover, in none of these studies agents' utilities are interdependent.

Among other related studies, Lai (2014) studies a communication game between an expert and an amateur who knows whether the state of the world is below or above a cutoff point that is her private information. He finds that the decision maker always ex-ante benefits from having access to the extra information. Ishida and Shimizu (2016)) analyze a communication game in which both the expert and the decision maker receive discrete imperfect signals about a binary state of the world. They show that when the two agents are equally informed, no information can be revealed in equilibrium for arbitrarily small biases. de Barreda (2010) investigates the strategic interaction between an informed expert and a decision maker when the latter has imperfect private information relevant to the decision.

4 Perfect Information Case

In this section, I assume the sender (CB) observes the state perfectly and sends a public signal to the private agents. Other than this signal, private agents have no other information on the state. Here, my goal is to look at how CB communicate with two players with complementarities otherwise it's the same model with Goltsman and Pavlov (2011).

Let us start with the first order conditions for the player i

$$\frac{\partial U_i}{\partial a_i} = -2(1-r)(a_i - \theta - b_i) - 2r(a_1 - a_2)$$

which in return gives the optimal action of the agent i as

$$a_i = (1-r)(\theta + b_i) + ra_j, \quad i, j = 1, 2. \tag{3}$$

Solving for the equilibrium gives player i's optimal action conditional on his information set:

$$a_i^* = E_i(\theta) + \hat{b}_i, \qquad i = 1, 2. \tag{4}$$

where E_i is the player i's information set, and $\hat{b}_i = \frac{1}{1+r}(b_i + rb_j), \quad (i, j = 1, 2)$.

Since there is only public information in this model, players' information sets are the same and this observation gives the following lemma.

5 Lemma *For any message $m(\theta)$, equilibrium actions satisfy*

$$a_1^* - a_2^* = \tilde{r}(b_1 - b_2),$$

where $\tilde{r} = \frac{1-r}{1+r}$.

Proof Directly follows from equation (4), and the observation that $E_1(\theta|m) = E_2(\theta|m)$ for any message m. ◊

Now the theorem characterizes the public equilibrium.

7 Theorem

In any public equilibrium the state space is partitioned into a finite number of points $\{\theta_0, \theta_1, \ldots, \theta_N\}$ where each point is given by the following expression

$$\theta_k = \frac{k}{N} + k(N-k)(b_1 + b_2), \quad k = 0, 1, \ldots, N$$

where $N \in \{1, \ldots, N(b_1 + b_2)\}$ with $N(b_1 + b_2) = \parallel \frac{-1}{2} + \frac{1}{2}(1 + \frac{4}{b_1+b_2})^{\frac{1}{2}} \parallel$.

Moreover, when $\theta \in (\theta_{k-1}, \theta_k)$ the equilibrium actions are given by

$$a_{i,k}(\theta) = \frac{1}{2}(\theta_{k-1} + \theta_k) + \frac{1}{1+r}(b_i + rb_j), \qquad i, j = 1, 2.$$

($\parallel x \parallel$ denotes the smallest integer greater than or equal to x.)

Comparing the equilibrium characterization given in Theorem 7 with the one in Crawford and Sobel's model gives immediately the following result.

8 Corollary *Public equilibrium partition is independent of the complementarity parameter r and the sender (CB) behaves as if she faces with a single receiver with the bias $(b_1 + b_2)/2$.*

This corollary allows us to interpret the theorem. Theorem basically says that CB divides the state space into a finite number of partitions, where the number of partitions is determined by the average bias of the receivers, and then CB sends the same message for every state in the same partition. Upon receiving the message, receivers take actions as if the true state is the middle point of the partition that the message belongs to.

One would expects that the complementarity would effect the characterization of the equilibria, but as the previous corollary states, it does not. The next corollary shows why CB does not take the complementarity into account when communicating with the private agents.

9 Corollary *In a public equilibrium with N partitions, the ex-ante expected utility of the sender (CB) and the players are given in the following expressions, respectively:*

$$\begin{aligned} EU_i &= -\frac{1}{12}\left[\frac{1}{N^2} + (b_1+b_2)^2(N^2-1)\right] \\ &= -\frac{r(b_i-b_j)}{1+r}\left[1 + \frac{(1-r)^2}{1+r}(b_i-b_j)\right], \quad i,j = 1,2. \\ EU_{CB} &= -\frac{1}{6}\left[\frac{1}{N^2} + (b_1+b_2)^2(N^2-1)\right] - b_1^2 - b_2^2 \end{aligned}$$

Proof See Appendix. ◊

5 Imperfect Information Case

The main purpose of this section is to characterize the equilibrium of the model given in the previous section as a function of the precision parameter, and then investigate how the social welfare (communication quality) changes as the precision of the public and private signals increase. However, this characterization might be very complicated as noted by the previous studies (see Chen (2009), de Barreda (2010)). Therefore, rather than giving a complete characterization of the equilibria they usually focus on monotone equilibria (a la Crawford and Sobel) of the game. Here, I also follow a similar approach.

In fact, in this kind of environments communication naturally, involves one more stage in which the public authority (CB) has the ability to collect private agents' opinions about the state of the nature before making a public announcement. This issue has been discussed by Chen. She finds that no information can be transmitted in this stage since the whole purpose of the receiver is to extract more information from the sender which in turn prevents any information transmission from receiver to the sender in this initial stage. However, since in my model there are two private agents and their utilities are complementary, they might prefer to reveal some of the information they possess to the CB in order to be able to better coordinate around the true state through the public announcement. Therefore, to add this initial stage to the model might be interesting.

In the rest of this section, I am going to look at how the precision of the public signal affect the spectrum of audiences she can credibly talk to. This exercise is an extension of Chen (2009) to two receivers case. I assume both CB and the players receive some informative but noisy signals on the state of nature. In order to concentrate on how the difference between precisions of the signals of CB and players affect communication quality, I work in a simple environment where the state θ is a binary variable with a diffuse prior and the players receive two different signals with equal precisions. More formally,

- $\theta \in \{0, 1\}$
- Prior on θ: $p\{\theta = 0\} = p\{\theta = 1\} = 1/2$
- CB's signal, y: $p\{y = t|\theta = t\} = p, \qquad t = 0, 1$
- Player i's signal, x_i: $p\{x_i = t|\theta = t\} = q, \qquad t = 0, 1 \quad i = 1, 2$

Here $p, q \in (1/2, 1]$ denotes precisions of the public and the private information, respectively.

First I consider the case that each player i keeps her signal, $\{x_i\}$ private and CB makes an announcement based on just her information y. I am interested in how *the range of the truth-telling equilibria* changes in the precision of CB's signal, p. Note that in equation (4), optimal action depends on the expected value of the state, where expectation is taken over the respective information set. In this section, as player i has his own information as well as a message from CB, his information set is $\{y, x_i\}$. Accordingly, let us denote the optimal action of agent i when CB's signal is k and his own signal is l by $a_i(k, l)$ and let $P_{k|l} = Prob(x_i = k \mid y = l)$ denote the probability that player i observes signal k when CB observes l, $k, l \in \{0, 1\}$.

Before stating the main theorem of this section, the following lemma proves useful.

10 Lemma $E\big[(a(x,y) - \theta)^2 \mid x, y\big] = \big[(a(x,y) - E(\theta \mid x, y)\big]^2 + var(\theta \mid x, y)$

Proof See Appendix. ◊

The theorem shows that for a given level of the average bias, CB's ability to announce the true state depends on precisions of both CB's and private agents' signals, and vice versa. Similar to the results of the first section, CB only cares about the average bias and ignores the complementarity among the agents. But here signal precision of the private agents also affects the existence of truthful equilibria as well as the precision of CB's own signal.

11 Theorem

In the imperfect information case, there is a threshold value b^* for the average bias, which is given by

$$b^* = q(1-q)(2p-1)\left[\frac{1}{P_{0|1}P_{1|1}} - 2\right]$$

such that CB can truthfully report her signal only when $b_1 + b_2 \leq b^*$.

Proof Since CB has downward bias, upon observing $y = 0$, CB has no incentive to misreport her signal. Therefore, in order to characterize the truth-telling equilibria it is sufficient to concentrate on the case when CB's signal is 1. So , suppose that CB observes $y = 1$. In this case, the incentive constraint of CB to tell the truth is

$$EU_{CB}[a_1(1, x_1), a_2(1, x_2), \theta \mid y = 1] \geq EU_{CB}[a_1(0, x_1), a_2(0, x_2), \theta \mid y = 1], \quad (IC_1)$$

IC_1 can be simplified further as follows.

$$\begin{aligned}
&E\big[-\big(a_1(1, x_1) - \theta\big)^2 - \big(a_2(1, x_2) - \theta\big)^2 \mid y = 1\big] \\
&\geq E\big[-\big(a_1(0, x_1) - \theta\big)^2 - \big(a_2(0, x_2) - \theta\big)^2 \mid y = 1\big] \\
&\Leftrightarrow \sum_{k=0}^{1} P_{k|1} E_{1k}\big[a_1(1, k) - \theta\big]^2 + \sum_{k=0}^{1} P_{k|1} E_{1k}\big[a_2(1, k) - \theta\big]^2 \\
&\leq \sum_{k=0}^{1} P_{k|1} E_{1k}\big[a_1(0, k) - \theta\big]^2 + \sum_{k=0}^{1} P_{k|1} E_{1k}\big[a_2(0, k) - \theta\big]^2
\end{aligned}$$

where $E_{lk}(\cdot) = E[(\cdot) \mid y = l, x = k]$.

First note that using the optimal action given in equation (4) and Lemma 10 we obtain

$$\begin{aligned}
\sum_{k=0}^{1} P_{k|1} E_{1k}\left[a_i(1, k) - \theta\right]^2 &= \sum_{k=0}^{1} P_{k|1}\left[E_{11}(\theta) + \tfrac{1}{1+r}(b_i + rb_j) - E_{11}(\theta)\right]^2 \\
&= \left(\tfrac{1}{1+r}(b_i + rb_j)\right)^2 \qquad i, j = 1, 2.
\end{aligned}$$

Then, the left hand side of IC_1 becomes

$$\left(\frac{1}{1+r}(b_1 + rb_2)\right)^2 + \left(\frac{1}{1+r}(b_2 + rb_1)\right)^2 = \hat{b}_1^2 + \hat{b}_2^2$$

Using Lemma 10, the right-hand side of IC_1 can be written as

$$\begin{aligned}
&P_{0|1}\big(E(\theta|0, 0) - E(\theta|1, 0) + \hat{b}_1\big)^2 + P_{1|1}\big(E(\theta|0, 1) - E(\theta|1, 1) + \hat{b}_1\big)^2 \\
&+ P_{0|1}\big(E(\theta|0, 0) - E(\theta|1, 0) + \hat{b}_2\big)^2 + P_{1|1}\big(E(\theta|0, 1) - E(\theta|1, 1) + \hat{b}_2\big)^2
\end{aligned}$$

For convenience I define

$$x_i = E(\theta|0,i) - E(\theta|1,i), \quad i = 1,2.$$

Then IC_1 can be expressed as

$$\begin{aligned}
(\hat{b}_1^2 + \hat{b}_2^2) &\leq P_{0|1}(x_0 + \hat{b}_1)^2 + P_{1|1}(x_1 + \hat{b}_1)^2 + P_{0|1}(x_0 + \hat{b}_1)^2 + P_{1|1}(x_1 + \hat{b}_1)^2 \\
&\Leftrightarrow P_{0|1}\big((x_0 + \hat{b}_1)^2 + (x_0 + \hat{b}_2)^2\big) + P_{1|1}\big((x_1 + \hat{b}_1)^2 + (x_1 + \hat{b}_2^2)\big) \\
&\Leftrightarrow P_{0|1}\big(2x_0^2 + 2x_0(\hat{b}_1 + \hat{b}_2) + (\hat{b}_1^2 + \hat{b}_2^2)\big) + P_{1|1}\big(2x_1^2 + 2x_1(\hat{b}_1 + \hat{b}_2) + (\hat{b}_1^2 + \hat{b}_2^2)\big) \\
&\Leftrightarrow 2P_{0|1}\big(x_0^2 + x_0(\hat{b}_1 + \hat{b}_2)\big) + 2P_{1|1}\big(x_1^2 + x_1(\hat{b}_1 + \hat{b}_2)\big) + (\hat{b}_1^2 + \hat{b}_2^2) \\
0 &\leq P_{0|1}\big(x_0^2 + x_0(\hat{b}_1 + \hat{b}_2)\big) + P_{1|1}\big(x_1^2 + x_1(\hat{b}_1 + \hat{b}_2)\big) \\
b_1 + b_2 &\leq -\left[\frac{P_{0|1}x_0^2 + P_{1|1}x_1^2}{P_{0|1}x_0 + P_{1|1}x_1}\right] \\
&\leq -\left[\frac{(P_{0|1}x_0 + P_{1|1}x_1)^2 - 2(P_{0|1}P_{1|1}x_0x_1)}{P_{0|1}x_0 + P_{1|1}x_1}\right] \\
&\leq -\left[P_{0|1}x_0 + P_{1|1}x_1 - 2\frac{P_{0|1}x_xP_{1|1}x_1}{P_{0|1}x_0 + P_{1|1}x_1}\right]
\end{aligned}$$

Before proceeding note that

$$\begin{aligned}
E(\theta|0,0) &= \frac{(1-p)(1-q)}{P_{1|1}}, \qquad & E(\theta|1,0) &= \frac{p(1-q)}{P_{0|1}} \\
E(\theta|0,1) &= \frac{(1-p)q}{P_{0|1}}, & E(\theta|1,1) &= \frac{pq}{P_{1|1}},
\end{aligned}$$

where $P_{0|1} = (p+q) - 2pq$ and $P_{1|1} = 2pq + 1 - (p+q)$.

Now I can calculate $P_{0|1}x_0$ and $P_{1|1}x_1$ as follows

$$\begin{aligned}
P_{0|1}x_0 &= (1-q)\left[\frac{1-p}{P_{1|1}-\frac{p}{P_{0|1}}}\right]P_{0|1} \\
&= (1-q)\left[\frac{P_{0|1}(1-p)-pP_{1|1}}{P_{1|1}}\right] \\
&= \frac{(1-q)}{P_{1|1}}\left[P_{0|1}-p\underbrace{(P_{0|1}+P_{1|1})}_{=1}\right] \\
&= \frac{(1-q)}{P_{1|1}}[p+q-2pq-p] \\
&= \frac{q(1-q)(1-2p)}{P_{1|1}} \\
P_{1|1}x_1 &= q\left[\frac{1-p}{P_{0|1}}-\frac{p}{P_{1|1}}\right]P_{1|1} \\
&= q\left[\frac{(1-p)P_{1|1}-pP_{0|1}}{P_{0|1}}\right] \\
&= \frac{q(1-q)(1-2p)}{P_{0|1}}
\end{aligned}$$

and consequently the expressions that we need becomes

$$\begin{aligned}
P_{0|1}x_0+P_{1|1}x_1 &= q(1-q)(1-2p)\left[\frac{1}{P_{0|1}}+\frac{1}{P_{1|1}}\right] \\
&= q(1-q)(1-2p)\frac{1}{P_{0|1}P_{1|1}} \\
P_{0|1}x_0P_{1|1}x_1 \quad &= [q(1-q)(1-2p)]^2\frac{1}{P_{0|1}P_{1|1}}
\end{aligned}$$

Finally, IC_1 reduces to the following expression as desired

$$b_1+b_2 \leq q(1-q)(2p-1)\left[\frac{1}{P_{0|1}P_{1|1}}-2\right]$$

◊

Next I present the main comparative statistics result of the paper which states that the ranges of average bias for which a truth-telling equilibrium exists is increasing in the precision of CB's signal and it is independent of the realizations of the players' signals. This result can be interpreted as CB increases her precision, the range of audiences to whom she can credibly communicate increases.

12 Corollary *The range of the average bias that CB can truthfully communicate with the private agents increases in the precision of her signal, i.e.*

$$\frac{\partial b^*}{\partial p} > 0\,.$$

Proof For the proof, it is enough to observe that $\frac{\partial P_{0|1}P_{1|1}}{\partial p} < 0$ as the following direct calculation shows

$$\begin{aligned}\frac{\partial P_{0|1}P_{1|1}}{\partial p} &= (1-2q)(2qp+1-p-q)+(p+q-2pq)(2q-1)\\ &= (2q-1)\big(2(p+q)-4pq-1\big) < 0, \quad \text{for} \quad p,q \in (1/2,1]\end{aligned}$$

6 Conclusions

In this chapter, I studied a model in which a central bank (CB) communicates with two private agents when all three players have different interests over the state of the economy and receive informative but noisy signals when the state is realized. I find that CB only cares about the average bias of the private agents, and CB can truthfully reveal her signal only if the average bias is below a certain threshold. Moreover, as CB's signal precision increases, the possibility of truthful communication increases in her signal precision which in return enhances her ability to communicate with the agents.

Appendix to Chapter 3

Proof of Theorem 1.

First I rewrite CB's utility function as

$$U_{CB} = -2\left(\frac{a_1 + a_2}{2} - \theta\right)^2 - 2\left(\frac{a_1 - a_2}{2}\right)^2$$

Moreover, using Lemma 5 allows to express this utility function just as a function of first player's action

$$\begin{aligned} U_{CB} &= -2\left(\frac{a_1 + a_1 - \tilde{r}(b_1 - b_2)}{2} - \theta\right)^2 - 2\left(\tilde{r}(b_1 - b_2)\right)^2 \\ &= -2\left(a_1 - \theta - \frac{\tilde{r}(b_1 - b_2)}{2}\right)^2 - 2\left(\tilde{r}(b_1 - b_2)\right)^2 \end{aligned}$$

Considering this equation and the equilibrium best response of player 1 shows that in any equilibrium only a finite number of actions can be induced, just as in Crawford and Sobel's model, and equilibrium is partitional.

Let $\{\theta_1, \theta_2, \ldots, \theta_N\}$ be a partition of $[0, 1]$. Indifference condition for type θ_k between actions $a_{1,k}$ and $a_{1,k+1}$ is given by

$$-2\left(a_{k,1} - \theta_k - \frac{\tilde{r}(b_1 - b_2)}{2}\right)^2 = -2\left(a_{1,k+1} - \theta_k - \frac{\tilde{r}(b_1 - b_2)}{2}\right)^2$$

which implies

$$\theta_k = \frac{a_k + a_{k+1}}{2} + \frac{\tilde{r}(b_1 - b_2)}{2}$$

Upon observing a message $m \in (\theta_{k-1}, \theta_k)$, player 1 take the equilibrium action

$$a_{1,k}(\theta) = \frac{1}{2}\left(\theta_{k-1} + \theta_k\right) + \frac{1}{1 + r}\left(b_1 + rb_2\right)$$

Combining last two equations characterizes the cut-off points as a second-order difference equation

$$\begin{aligned}\theta_{k+1} &= 2\theta_k - \theta_{k-1} - \frac{4}{1+r}(b_1 + rb_2) - \frac{4\tilde{r}}{2}(b_2 - b_1) \\ &= 2\theta_k - \theta_{k-1} - 2(b_1 + b_2)\end{aligned}$$

This second-order linear difference equation has a class of solutions parameterized by θ_1 (given that $\theta_0 = 0$)

$$\theta_k = k\theta_1 - k(k-1)(b_1 + b_2)$$

To have a solution we must impose $|-k(k-1)(b_1+b_2)| < 1$, which is satisfied when

$$k \leq \| \frac{-1}{2} + \frac{1}{2}\left(1 + \frac{4}{b_1 + b_2}\right)^{\frac{1}{2}} \|$$

This is the number given as $N(b_1 + b_2)$ in the theorem.

Now using the boundary condition $\theta_N = 1$ determines $\theta_1 = \frac{1}{N} + (N-1)(b_1 + b_2)$, and finally substituting this into the previous expression gives boundary points as

$$\theta_k = \frac{k}{N} + k(N-k)(b_1 + b_2), \quad k = 0, 1, \ldots, N.$$

Proof of Corollary 4.

$$\begin{aligned}EU_1 &= -(1-r)E(a_1 - \theta - b_1)^2 - rE(a_1 - a_2)^2 \\ &= -(1-r)E(a_1 - \theta - b_1)^2 - r(\tilde{r}(b_1 - b_2))^2 \quad \text{(Lemma 5)}\end{aligned}$$

Now I compute the first term

$$
\begin{aligned}
-E(a_1-\theta-b_1)^2 &= -\sum_{k=1}^{N}\int_{\theta_{k-1}}^{\theta_k}(a_{1,k}-\theta-b_1)^2 d\theta \\
&= -\sum_{k=1}^{N}\int_{\theta_{k-1}}^{\theta_k}\left(\frac{1}{2}(\theta_{k-1}+\theta_k)+\frac{1}{1+r}(b_1+rb_2)-\theta-b_1\right)^2 d\theta \\
&= -\sum_{k=1}^{N}\int_{\theta_{k-1}}^{\theta_k}\left[\frac{1}{2}(\theta_{k-1}+\theta_k)-\theta-\frac{r}{1+r}(b_1-b_2)\right] d\theta \\
&= \frac{1}{3}\sum_{k=1}^{N}\left[\frac{1}{2}(\theta_{k-1}+\theta_k)-\theta-b\right]^3 |_{\theta_{k-1}}^{\theta_k} \quad (b=\frac{r}{1+r}(b_1-b_2)) \\
&= \frac{1}{3}\sum_{k=1}^{N}\left[\frac{1}{2}(\theta_{k-1}-\theta_k)-b\right]^3-\left[\frac{1}{2}(\theta_{k-1}+\theta_k)-b\right]^3 \\
&= \frac{1}{3}\sum_{k=1}^{N}\left[(-x-b)^3-(x-b)^3\right], \quad (x=\frac{1}{2}(\theta_{k-1}-\theta_k)) \\
&= -\frac{1}{3}\sum_{k=1}^{N}2x^3+6xb^2
\end{aligned}
$$

Using the expression for θ_k in the Theorem 1, I get

$$
\begin{aligned}
x &= \frac{1}{2}\left[\theta_{k-1}-\theta_k\right] \\
&= \frac{1}{2}\left[\frac{1}{N}+(b_1+b_2)(1+N-2k)\right]
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\sum 2x^3 &= \frac{1}{4}\left[N^{-2}+(b_1+b_2)^2(N^2-1)\right] \\
\sum 6xb^2 &= 6\frac{1}{2}\sum\frac{1}{N}+(b_1+b_2)(1+N-2k) \\
&= 3b^2
\end{aligned}
$$

Now I obtain,

$$
\begin{aligned}
EU_1 &= \frac{-(1-r)}{3}\left[\frac{1}{4}[N^{-2} + (b_1+b_2)^2(N^2-1)] + 3b^2\right] - r(\tilde{r}(b_1-b_2))^2 \\
&= -\frac{(1-r)}{12}\left[\frac{1}{N^2} + (b_1+b_2)^2(N^2-1)\right] - \frac{r(b_1-b_2)}{1+r}\left[1 + \frac{(1-r)^2}{1+r}(b_1-b_2)\right]
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
EU_{CB} &= -E(a_1-\theta)^2 - E(a_2-\theta)^2 \\
&= -E(a_1-\theta-b_1)^2 - E(a_2-\theta-b_2)^2 - b_1^2 - b_2^2 \\
&= -\frac{1}{6}\left[\frac{1}{N^2} + (b_1+b_2)^2(N^2-1)\right] - b_1^2 - b_2^2
\end{aligned}
$$

Proof of Lemma 5.

$$
\begin{aligned}
E\left[(a(x,y)-\theta)^2 \mid x,y\right] &= E\left[(a(x,y) - E(\theta \mid x,y) + E(\theta \mid x,y) - \theta)^2 \mid x,y\right] \\
&= E\left[\left(a - E_{yx}(\theta)\right)^2 + E(\theta - E_{yx}(\theta))^2\right] \\
&\quad -2E\left[(a - E_{yx}(\theta))(\theta - E_{yx}(\theta)) \mid x,y\right]\Big] \\
&= \left(a - E_{yx}(\theta)\right)^2 + var(\theta \mid x,y)
\end{aligned}
$$

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