Problem Set 4 - Solutions

Applied Statistics and Econometrics II, Spring 2018

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Contents

Problem 1

Problem 1.a.

$$\begin{aligned}\widehat{\beta} &= (Z'X)^{-1} Z'Y \\ &= \left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1} \left(\sum_{i=1}^{n} z_i y_i\right)\end{aligned}$$

Problem 1.b.

Write

$$\widehat{\beta} - \beta = \left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1} \left(\sum_{i=1}^{n} z_i e_i\right)$$

Then

$$
\begin{aligned}
E\left(\widehat{\beta}-\beta \mid Z, X\right) &= E\left[\left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1}\left(\sum_{i=1}^{n} z_i e_i\right) \mid Z, X\right] \\
&= \left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1}\left(\sum_{i=1}^{n} z_i E(e_i \mid Z, X)\right) \\
&= \left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1}\left(\sum_{i=1}^{n} z_i E(e_i \mid z_i, x_i)\right) \\
&= 0.
\end{aligned}
$$

Thus by the law of iterated expectations $E\left(\widehat{\beta}\right)=\beta$, i.e. $\widehat{\beta}$ is an unbiased estimator for β.

Problem 1.c.

Since $E\left(\widehat{\beta} \mid Z, X\right)=\beta$ we have

$$
\begin{aligned}
\operatorname{var}\left(\widehat{\beta} \mid Z, X\right) &= E\left(\left(\widehat{\beta}-\beta\right)\left(\widehat{\beta}-\beta\right)^{'} \mid Z, X\right) \\
&= E\left[\left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1}\left(\sum_{i=1}^{n} z_i e_i\right)\left(\sum_{i=1}^{n} e_i z_i^{'}\right)\left(\sum_{i=1}^{n} x_i z_i^{\prime}\right)^{-1} \mid Z, X\right] \\
&= \left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} E\left(z_i e_i e_j z_j^{'} \mid Z, X)\right)\right)\left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1} \\
&= \left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1}\left(\sum_{i=1}^{n} E\left(z_i e_i e_j z_j^{'} \mid z_i, x_i)\right)\right)\left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1} \\
&= \left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1}\left(\sum_{i=1}^{n} z_i z_i^{'} \sigma_i^2\right)\left(\sum_{i=1}^{n} z_i x_i^{'}\right)^{-1}
\end{aligned}
$$

where $\sigma_i^2 = E(e_i^2 \mid x_i, z_i)$. Notice that this is the variance conditional on both z and x

Problem 2

Discussed in class.

Problem 3

```
rm(list = ls())
dataP1 <- load("401ksubs.RData")
dataP1 <- data
```

Problem 3.a

```r
lmP1a <- lm(pira ~ p401k + inc + incsq + age + agesq, data=dataP1)
#or
lmP1a <- lm(pira ~ p401k + inc + I((inc)^2) + age + I((age)^2), data=dataP1)
summary(lmP1a)
```

```
##
## Call:
## lm(formula = pira ~ p401k + inc + I((inc)^2) + age + I((age)^2),
##     data = dataP1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.8855 -0.2629 -0.1198  0.1987  1.0625
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.977e-01  6.864e-02  -2.880  0.00398 **
## p401k        5.366e-02  9.571e-03   5.606 2.13e-08 ***
## inc          8.679e-03  5.110e-04  16.983  < 2e-16 ***
## I((inc)^2)  -2.280e-05  4.033e-06  -5.653 1.62e-08 ***
## age         -1.594e-03  3.330e-03  -0.479  0.63228
## I((age)^2)   1.173e-04  3.823e-05   3.068  0.00216 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3945 on 9269 degrees of freedom
## Multiple R-squared:   0.18,  Adjusted R-squared:  0.1795
## F-statistic: 406.9 on 5 and 9269 DF,  p-value: < 2.2e-16
```

The coefficient on p401k implies that participation in a 401(k) plan is associated with a .054 higher probability of having an individual retirement account, holding income and age fixed.

Problem 3.b

While the regression in part (a) controls for income and age, it does not account for the fact that different people have different taste for savings, even within given income and age categories. People that tend to be savers will tend to have both a 401(k) plan as well as an IRA. (This means that the error term, is positively correlated with p401k.) What we would like to know is, for a given person, if that person participates in a 401(k), does it make it less likely or more likely that the person also has an IRA. This ceteris paribus question is difficult to answer by OLS without many more controls for the taste for saving.

Problem 3.c

First, we need e401k to be partially correlated with p401k; not surprisingly, this is not an issue, as being eligible for a 401(k) plan is, by definition, necessary for participation. (The regression in part (d) verifies that they are strongly positively correlated.) The more difficult issue is whether e401k can be taken as exogenous in the structural model. In other words, is being eligible for a 401(k) correlated with unobserved taste for saving? If we think workers that like to save for retirement will match up with employers that provide vehicles for retirement saving, then the error term e and e401k would be positively correlated. Certainly we think that e401k is less correlated with e than is p401k. But remember, this alone is not enough to ensure that the IV estimator has less asymptotic bias than the OLS estimator.

Problem 3.d

The reduced form equation, estimated by OLS but with heteroskedasticity-robust standard errors:

```
lmP1d <- lm(p401k ~ e401k + inc + incsq + age + agesq, data=dataP1)
sumP1d <- summary(lmP1d)
sumP1d
```

```
##
## Call:
## lm(formula = p401k ~ e401k + inc + incsq + age + agesq, data = dataP1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.89568 -0.01371  0.01310  0.24283  0.34692
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.915e-02  4.951e-02   1.195  0.23228
## e401k        6.888e-01  6.351e-03 108.469  < 2e-16 ***
## inc          1.112e-03  3.706e-04   3.000  0.00271 **
## incsq        1.841e-06  2.918e-06   0.631  0.52810
## age         -4.721e-03  2.403e-03  -1.964  0.04952 *
## agesq        5.204e-05  2.759e-05   1.886  0.05927 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2842 on 9269 degrees of freedom
## Multiple R-squared:  0.5963, Adjusted R-squared:  0.5961
## F-statistic:  2738 on 5 and 9269 DF,  p-value: < 2.2e-16
```

Obtain heteroskedasticity-robust standard errors:

```
#install.packages("sandwich")
library(sandwich)
vcvP1d <- sqrt(diag(vcovHC(lmP1d)))
vcvP1d
```

```
##  (Intercept)        e401k          inc        incsq          age
## 4.621265e-02 7.993167e-03 3.449603e-04 2.694549e-06 2.244852e-03
##        agesq
## 2.573035e-05
```

```
tP1d <- coef(lmP1d)/vcvP1d
tP1d
```

```
## (Intercept)       e401k         inc       incsq         age       agesq
##   1.2799378  86.1792897   3.2225825   0.6832342  -2.1028154   2.0224136
```

The t statistic on e401k is over 85, and its coefficient estimate implies that, holding income and age fixed, eligibility in a 401(k) plan increases the probability of participation in a 401(k) by .69. Clearly, e401k passes one of the two requirements as an IV for p401k.

Problem 3.e

```r
#install.packages("ivpack")
library(ivpack)
```

```
## Loading required package: AER

## Loading required package: car

## Loading required package: carData

## Loading required package: lmtest

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: survival
```

```r
ivP1e <- ivreg(pira ~ p401k + inc + incsq + age + agesq,~e401k+inc+incsq+age+agesq,data=dataP1)
ivP1e
```

```
##
## Call:
## ivreg(formula = pira ~ p401k + inc + incsq + age + agesq | e401k +     inc + incsq + age + agesq, da
##
## Coefficients:
## (Intercept)        p401k          inc        incsq          age
##  -2.073e-01    2.070e-02    8.998e-03   -2.414e-05   -1.147e-03
##       agesq
##   1.121e-04
```

To obtain heteroskedasticity-robust standard errors either apply the procedure in part (d) or simply

```r
robust.se(ivP1e)
```

```
## [1] "Robust Standard Errors"

##
## t test of coefficients:
##
##                Estimate  Std. Error t value  Pr(>|t|)
## (Intercept) -2.0731e-01  6.5359e-02 -3.1719  0.001519 **
## p401k        2.0701e-02  1.3228e-02  1.5650  0.117624
## inc          8.9982e-03  4.9113e-04 18.3215 < 2.2e-16 ***
## incsq       -2.4136e-05  3.8809e-06 -6.2192 5.212e-10 ***
## age         -1.1466e-03  3.2480e-03 -0.3530  0.724071
## agesq        1.1207e-04  3.8324e-05  2.9243  0.003461 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The IV estimate of p401k is less than half as large as the OLS estimate, and the IV estimate has a t statistic roughly equal to 1.56. The reduction in p401k is what we expect given the unobserved taste for saving argument made in part (b). But we still do not estimate a tradeoff between participating in a 401(k) plan and participating in an IRA. This conclusion has prompted some in the literature to claim that 401(k) saving is additional saving; it does not simply crowd out saving in other plans.

Problem 4

```r
rm(list = ls())
dataP2 <- load("fertil2.RData")
dataP2 <- data
```

Problem 4.a

The equation estimated by OLS is

```r
lmP2a <- lm(children ~ educ + age + agesq, data=dataP2)
summary(lmP2a)
```

```
##
## Call:
## lm(formula = children ~ educ + age + agesq, data = dataP2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.8351 -0.7135 -0.0054  0.7141  7.5055
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.1383066  0.2405942 -17.200   <2e-16 ***
## educ        -0.0905755  0.0059207 -15.298   <2e-16 ***
## age          0.3324486  0.0165495  20.088   <2e-16 ***
## agesq       -0.0026308  0.0002726  -9.651   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.46 on 4357 degrees of freedom
## Multiple R-squared:  0.5687, Adjusted R-squared:  0.5684
## F-statistic:  1915 on 3 and 4357 DF,  p-value: < 2.2e-16
```

Another year of education, holding age fixed, results in about .091 fewer children. In other words, for a group of 100 women, if each gets another year of education, they collectively are predicted to have about nine fewer children.

Problem 4.b

The reduced form for educ is

```r
lmP2b <- lm(educ ~ frsthalf + age + agesq, data=dataP2)
summary(lmP2b)
```

```
##
## Call:
## lm(formula = educ ~ frsthalf + age + agesq, data = dataP2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.9599 -2.4941  0.2663  2.2663 14.9934
##
```

```
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  9.6928643  0.5980686  16.207  < 2e-16 ***
## frsthalf    -0.8522854  0.1128296  -7.554 5.12e-14 ***
## age         -0.1079504  0.0420402  -2.568   0.0103 *
## agesq       -0.0005056  0.0006929  -0.730   0.4657
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.711 on 4357 degrees of freedom
## Multiple R-squared:  0.1077, Adjusted R-squared:  0.107
## F-statistic: 175.2 on 3 and 4357 DF,  p-value: < 2.2e-16
```

When we run the regression, we obtain the coefficient of firsthalf as -0.85 with std error 0.113. Therefore, women born in the first half of the year are predicted to have almost one year less education, holding age fixed. The t statistic on frsthalf is over 7.5 in absolute value, and so the identification condition holds.

Problem 4.c

In this case, X and Z matrices should read as follows

```r
attach(dataP2)
ones <- rep(1,length(children))
X <- cbind(ones, educ, age, agesq)
Z <- cbind(ones, frsthalf, age, agesq)
b_IV <- solve(t(Z)%*%X)%*%t(Z)%*%children
b_IV
```

```
##               [,1]
## ones  -3.387805354
## educ  -0.171498916
## age    0.323605220
## agesq -0.002672276
```

```r
detach(dataP2)
```

or using ivpack

```r
#install.packages("ivpack")
library(ivpack)
ivP2c <- ivreg(children ~ educ + age + agesq, ~ frsthalf + age + agesq, data=dataP2)
ivP2c
```

```
##
## Call:
## ivreg(formula = children ~ educ + age + agesq | frsthalf + age +     agesq, data = dataP2)
##
## Coefficients:
## (Intercept)         educ          age        agesq
##   -3.387805    -0.171499     0.323605    -0.002672
```

The estimated effect of education on fertility is now much larger. Naturally, the standard error for the IV estimate is also bigger, about nine times bigger. This produces a fairly wide 95% CI for the coefficient of educ.

Problem 4.d

When we add electric, tv, and bicycle to the equation and estimate it by OLS, we obtain

```
lmP2d <- lm(children ~ educ + age + agesq + electric + tv + bicycle, data=dataP2)
summary(lmP2d)
```

```
##
## Call:
## lm(formula = children ~ educ + age + agesq + electric + tv +
##     bicycle, data = dataP2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.7781 -0.7205 -0.0169  0.7168  7.5645
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.3897837  0.2403173 -18.267  < 2e-16 ***
## educ        -0.0767093  0.0063526 -12.075  < 2e-16 ***
## age          0.3402038  0.0164417  20.692  < 2e-16 ***
## agesq       -0.0027081  0.0002706 -10.010  < 2e-16 ***
## electric    -0.3027293  0.0761869  -3.974 7.20e-05 ***
## tv          -0.2531443  0.0914374  -2.768  0.00566 **
## bicycle      0.3178950  0.0493661   6.440 1.33e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.448 on 4349 degrees of freedom
##   (5 observations deleted due to missingness)
## Multiple R-squared:  0.5761, Adjusted R-squared:  0.5755
## F-statistic: 984.9 on 6 and 4349 DF,  p-value: < 2.2e-16
```

The 2SLS (or IV) estimates are

```
#install.packages("ivpack")
library(ivpack)
ivP2d <- ivreg(children ~ educ + age + agesq + electric + tv + bicycle,
               ~ frsthalf + age + agesq + electric + tv + bicycle, data=dataP2)
ivP2d
```

```
##
## Call:
## ivreg(formula = children ~ educ + age + agesq + electric + tv +     bicycle | frsthalf + age + agesq
##
## Coefficients:
## (Intercept)         educ          age        agesq     electric
##   -3.591332    -0.163981     0.328145    -0.002722    -0.106531
##          tv      bicycle
##   -0.002555     0.332072
```

Adding electric, tv, and bicycle to the model reduces the estimated effect of educ in both cases, but not by too much. In the equation estimated by OLS, the coefficient on tv implies that, other factors fixed, four families that own a television will have about one fewer child than four families without a TV. Television ownership can be a proxy for different things, including income and perhaps geographic location. A causal interpretation is that TV provides an alternative form of recreation.

Interestingly, the effect of TV ownership is practically and statistically insignificant in the equation estimated by IV (even though we are not using an IV for tv). The coefficient on electric is also greatly reduced in magnitude in the IV estimation. The substantial drops in the magnitudes of these coefficients suggest that a linear model might not be the best functional form, which would not be surprising since children is a count variable.

Problem 5

```r
rm(list = ls())
#install.packages("readxl")
library("readxl")
dataP3 <- read_excel("JEC.xls")
```

Problem 5.a

```r
lmP3a <- lm(log(quantity) ~ log(price) + ice +. - week - cartel - price, data=dataP3)
summary(lmP3a)
```

```
##
## Call:
## lm(formula = log(quantity) ~ log(price) + ice + . - week - cartel -
##     price, data = dataP3)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.39102 -0.24296  0.06575  0.28284  1.05884
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.861233   0.171361  51.711  < 2e-16 ***
## log(price)  -0.638885   0.082389  -7.755 1.26e-13 ***
## ice          0.447754   0.119604   3.744 0.000216 ***
## seas1       -0.132822   0.110959  -1.197 0.232197
## seas2        0.066888   0.111298   0.601 0.548286
## seas3        0.111436   0.111308   1.001 0.317527
## seas4        0.155422   0.110743   1.403 0.161477
## seas5        0.109658   0.129918   0.844 0.399282
## seas6        0.046833   0.159596   0.293 0.769377
## seas7        0.122552   0.160041   0.766 0.444397
## seas8       -0.235008   0.159856  -1.470 0.142533
## seas9        0.003561   0.160021   0.022 0.982262
## seas10       0.169247   0.161295   1.049 0.294849
## seas11       0.215184   0.160096   1.344 0.179890
## seas12       0.219633   0.159136   1.380 0.168524
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3973 on 313 degrees of freedom
## Multiple R-squared:  0.3126, Adjusted R-squared:  0.2819
## F-statistic: 10.17 on 14 and 313 DF,  p-value: < 2.2e-16
```

The estimated value of the demand elasticity is 0.64 and its standard error is 0.082.

Problem 5.b

Because of the simultaneity problem that we discussed in the class.

Problem 5.c

Relevance: it's likely that cartel is a relevant variable since being a cartel a firm can effect the prices by controling the supply.

Exogeneity: this part is not as obvious as the relevance. We can still assume that in equilibrium demand will be equal to supply and the supply of grain will be exogenously determined. However, note that this line of reasoning presupposes that this is an aggregate market.

Problem 5.d

The reduced form equation estimation:

```r
lmP3d <- lm(log(price) ~ cartel + ice +. - week - price - quantity, data=dataP3)
summary(lmP3d)
```

```
##
## Call:
## lm(formula = log(price) ~ cartel + ice + . - week - price - quantity,
##     data = dataP3)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.49765 -0.13625  0.01362  0.13616  0.55689
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.693741   0.078361 -21.615  < 2e-16 ***
## cartel       0.357898   0.024862  14.395  < 2e-16 ***
## ice          0.035003   0.064252   0.545  0.58629
## seas1        0.038725   0.059084   0.655  0.51268
## seas2        0.136288   0.059084   2.307  0.02173 *
## seas3        0.189049   0.059319   3.187  0.00158 **
## seas4        0.089523   0.059357   1.508  0.13251
## seas5        0.017863   0.069869   0.256  0.79838
## seas6       -0.025741   0.085529  -0.301  0.76364
## seas7       -0.067126   0.085529  -0.785  0.43314
## seas8       -0.035837   0.085709  -0.418  0.67614
## seas9       -0.005776   0.086321  -0.067  0.94670
## seas10      -0.100211   0.086321  -1.161  0.24656
## seas11      -0.086751   0.085362  -1.016  0.31028
## seas12       0.011693   0.085362   0.137  0.89113
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2114 on 313 degrees of freedom
## Multiple R-squared:  0.4881, Adjusted R-squared:  0.4652
```

```
## F-statistic: 21.32 on 14 and 313 DF,  p-value: < 2.2e-16
```

The coefficient of cartel is approximately 0.36 and it's highly significant, so we can't say that it's a weak instrument.

Problem 5.e

In this case, X and Z matrices should read as follows

```
attach(dataP3)
ones <- rep(1,length(price))
X<-cbind(ones,log(price),ice,seas1,seas2,seas3,seas4,seas5,seas6,seas7,seas8,seas9,seas10,seas11,seas12)
Z<-cbind(ones,cartel,ice,seas1,seas2,seas3,seas4,seas5,seas6,seas7,seas8,seas9,seas10,seas11,seas12)
b_IV <- solve(t(Z)%*%X)%*%t(Z)%*%log(quantity)
b_IV
```

```
##                [,1]
## ones    8.573534698
##        -0.866586589
## ice     0.422933926
## seas1  -0.130973232
## seas2   0.090952099
## seas3   0.135871955
## seas4   0.152510743
## seas5   0.073561727
## seas6  -0.006064189
## seas7   0.060232198
## seas8  -0.293599206
## seas9  -0.058372353
## seas10  0.085810726
## seas11  0.151791011
## seas12  0.178655730
```

```
detach(dataP3)
```

or using ivpack

```
#install.packages("ivpack")
library(ivpack)
ivP3e <- ivreg(log(quantity) ~ log(price) + ice +. - week - cartel - price,
               ~cartel + ice +. - week - price, data=dataP3)
summary(ivP3e)
```

```
##
## Call:
## ivreg(formula = log(quantity) ~ log(price) + ice + . - week -
##     cartel - price | cartel + ice + . - week - price, data = dataP3)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.38295 -0.27275  0.07318  0.27703  1.09320
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.573535   0.216445  39.611  < 2e-16 ***
## log(price)  -0.866587   0.132123  -6.559 2.24e-10 ***
```

```
## ice          0.422934   0.121569   3.479 0.000575 ***
## seas1       -0.130973   0.112307  -1.166 0.244420
## seas2        0.090952   0.113167   0.804 0.422181
## seas3        0.135872   0.113194   1.200 0.230912
## seas4        0.152511   0.112094   1.361 0.174632
## seas5        0.073562   0.132494   0.555 0.579148
## seas6       -0.006064   0.163277  -0.037 0.970397
## seas7        0.060232   0.164392   0.366 0.714319
## seas8       -0.293599   0.163930  -1.791 0.074259 .
## seas9       -0.058372   0.164343  -0.355 0.722689
## seas10       0.085811   0.167514   0.512 0.608831
## seas11       0.151791   0.164530   0.923 0.356941
## seas12       0.178656   0.162119   1.102 0.271306
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4021 on 313 degrees of freedom
## Multiple R-Squared: 0.2959,  Adjusted R-squared: 0.2644
## Wald test: 8.807 on 14 and 313 DF,  p-value: 3.5e-16
```

The estimated value of the demand elasticity is -0.87 (-0.64) and its standard error is 0.13 (0.082), where the values in paranthesis are OLS results. However, the price elasticity of demand must be less than negative one for a monopoly.